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Capital Markets

Investment Knowledge: Mainstream and frontier concepts with a special focus on essentials of quantitative investment analysis
from the point of view of a qualified investment professional

Strategic project of TBU in Zlín, reg. no. CZ.02.2.69/0.0/0.0/16_015/0002204

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1. What are the mainstream and frontier concepts nowadays generally accepted and applied by investment professionals?



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1.1 What do we mean if we say “investment management industry” and “qualified investment professional”?

- According to KPMG (2014): “The *investment management industry* is a vibrant market place where participants scour the globe for sources of wealth and attractive opportunities in traditional and alternative investments.”
- According to FINRA (2019): “ A *qualified investment professional* can help to make investment decisions, assist with financial goal setting and keep informed about how the economy and financial markets are affecting investment portfolio. There are many types of investment professionals—including brokers, investment advisers and financial planners.”





1.1.1 What are the opportunities and challenges the investment management industry currently faces?

- **steady recovery** of the investment management industry since the 2008 financial crisis: *with the value of assets under management (“AUM”) reaching almost 80 trillion USD* (Statista, 2017)
- **regulatory change** particularly in US and Europe
- **new debate in the industry** formerly the switched the focus of CEOs and Boards from the survival in the short term to make their businesses sustainable growth, over the next 10 to 20 years; and nowadays with a core question how megatrends such as demographics, technology or the environment will impact business in the future.





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1.2 What do we mean if we say “GBIC®”?

- GBIC® means Global Body of Investment Knowledge

It is a comprehensive **outline of knowledge** for the **investment profession** and **includes actual mainstream and frontier concepts** ensured by the Education Advisory Committee (EAC) of CFA Institute (regularly updated every five years from 1995).

CFA Institute is the core global association for investment management professionals.



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1.2.1 Current mainstream and frontier concepts widely accepted as crucial for Investment Knowledge (GBIC®)

- I. Ethical and professional standards
- II. Quantitative methods
- III. Economics
- IV. Financial reporting and analysis
- V. Corporate finance
- VI. Equity investments
- VII. Fixed income
- VIII. Derivatives
- IX. Alternative investments
- X. Portfolio management and wealth planning

*What I need to know to
become a qualified
investment
professional or
financial adviser?*





I. Ethical and professional standards

- Ethical and professional standards are one of the major knowledge areas needed to be learned to become a qualified investment professional or financial adviser.
- According to CFA (2019) 3 main areas are highlighted as follows:

A. Applicable laws and regulations

consisting of 6 modules, ie 1) Governmental institutions, 2) Securities and Exchange Commission and equivalent bodies in other countries, 3) rules and procedures regarding corporate transactions and insider trading, 4)

B. Professional standards of practice

consisting of 2 modules: 1) The Code of Ethics, 2) Standards of Professional Conduct

C. Ethical practices and guidelines

consisting of 8 modules, ie. 1) Corporate Government, 2) Soft Dollar Standards, 3) Fiduciary Duty, 4) Insider Trading, 5) Personal Investing, 6) Research objectivity Standards, 7) Trade Management Guidelines, 8) Analyst/ Corporate Relations Guidelines.





II. Quantitative methods (for quantitative investment analysis)

- Quantitative methods are one of the major knowledge areas needed to be learned to become a qualified investment professional or financial adviser.

According to CFA (2019) the following areas are highlighted:

- Time value of money
- Basic statistical and probability concepts
- Probability distributions
- Sampling and estimations
- Hypothesis testing
- Correlation analysis and regression
- Time series analysis
- Simulation analysis
- Scenario & sensitivity analysis
- Technical analysis





III. Economics (for investment decision makers)

- Economics is one of the major knowledge areas needed to be learned to become a qualified investment professional or financial adviser.

According to CFA (2019) the following areas are highlighted:

- Market forces of supply and demand
- The firm and industry organization
- Measuring national income and growth
- Business cycles
- The monetary system & Inflation
- International trade and capital flows
- Currency exchange rates
- The macroeconomics of an open economy
- Aggregate demand and aggregate supply
- Economic growth and development
- Government regulation & relationship of economic activity to the investment process





IV. Financial reporting and analysis (international financial statement analysis)

- Financial reporting and analysis is one of the major knowledge areas needed to be learned to become a qualified investment professional or financial adviser.

According to CFA (2019) the following areas are highlighted:

- Financial reporting system & Financial reporting quality
- Principal financial statements
- Analysis of inventories
- Analysis of long-lived assets
- Analysis of taxes
- Analysis of debt
- Analysis of off-balance-sheet assets and liabilities
- Analysis of pensions, stock compensation, and other employee benefits
- Analysis of inter-corporate investments
- Analysis of business combination
- Analysis of global operations, of financial instruments, derivatives and hedging activities
- Financial reporting in specialized industries and economic environments





V. Corporate finance (& portfolio management)

- Corporate finance is one of the major knowledge areas needed to be learned to become a qualified investment professional or financial adviser.

According to CFA (2019) the following areas are highlighted:

- Fundamental issues
- Corporate governance
- Capital investment decisions
- Business and financial risk
- Long-term & short-term financial policy
- Dividend policy
- Mergers and acquisitions and corporate restructuring
- Valuation implications of corporate finance
- Introduction to investment banking
- Other corporate actions such as bonuses, splits and treasury stocks





VI. Equity investments (& equity asset valuation)

- Equity investments and equity asset valuation is one of the major knowledge areas needed to be learned to become a qualified investment professional or financial adviser.

According to CFA (2019) the following areas are highlighted:

- Types of equity securities and their characteristics
- Equity markets: characteristics, institutions, and benchmarks
- Equity market valuation and return analysis
- Fundamental analysis (sector, industry, company) and the valuation of individual equity securities
- Special applications of fundamental analysis





VII. Fixed income (& fixed income analysis)

- Fixed income and fixed income analysis is one of the major knowledge areas needed to be learned to become a qualified investment professional or financial adviser.

According to CFA (2019) the following areas are highlighted:

- Fixed income securities and their characteristics
- Fixed income markets: characteristics, institutions and benchmarks
- Fixed income valuation and return analysis
- Term structure determination and yield spreads
- Analysis of interest rate risk
- Analysis of bond risk
- Valuing bonds with embedded options
- Structured products and valuing
- Trading strategies and their assessment
- -Influence of equity market changes on bond pricing
- Behavioral issues





VIII. Derivatives

- Derivatives is one of the major knowledge areas needed to be learned to become a qualified investment professional or financial adviser.

According to CFA (2019) the following areas are highlighted:

- Types of derivative instruments and their characteristics
- Forward markets and instruments
- Futures markets and instruments
- Options markets and instruments
- Swaps markets and instruments
- Credit derivatives markets and instruments
- Other derivatives issues





IX. Alternative investments

- Alternative investments is one of the major knowledge areas needed to be learned to become a qualified investment professional or financial adviser.

According to CFA (2019) the following areas are highlighted:

- Types of alternative investment and their characteristics
- Real estate
- Investment companies
- Private equity
- Venture capital
- Hedge funds
- Closely-held companies and inactively traded securities
- Distressed securities/bankruptcies
- Commodities
- Tangible assets with low liquidity





X. Portfolio management and wealth planning (1)

- Portfolio management and wealth planning is one of the major knowledge areas needed to be learned to become a qualified investment professional or financial adviser.

According to CFA (2019) the following areas are highlighted:

- Portfolio concepts from capital market theory
- Management of individual/family investor portfolios
- Management of institutional investor portfolios
- Pension plans and employee benefit funds
- Endowment funds and foundations
- Insurance companies
- Mutual funds, pooled funds, and ETFs
- Other institutional investors (banks, non-financial corporations)
- Investment manager selection





X. Portfolio management and wealth planning (2)

According to CFA (2019) the following areas are highlighted (cont.):

- Economic analysis and capital market expectations
- Asset allocation
- Portfolio construction and revision
- Equity portfolio management strategies
- Fixed income portfolio management strategies
- Alternative investment portfolio management strategies
- Risk management
- Execution of portfolio decisions (trading)
- Performance measurement, attribution and appraisal
- Presentation of performance results





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2. Case study – Global management services in the accounting Big Four

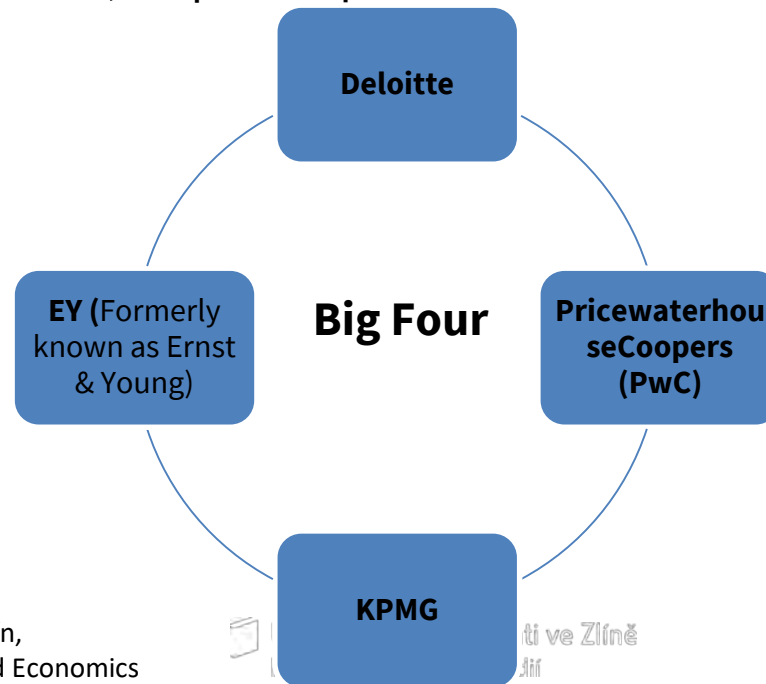


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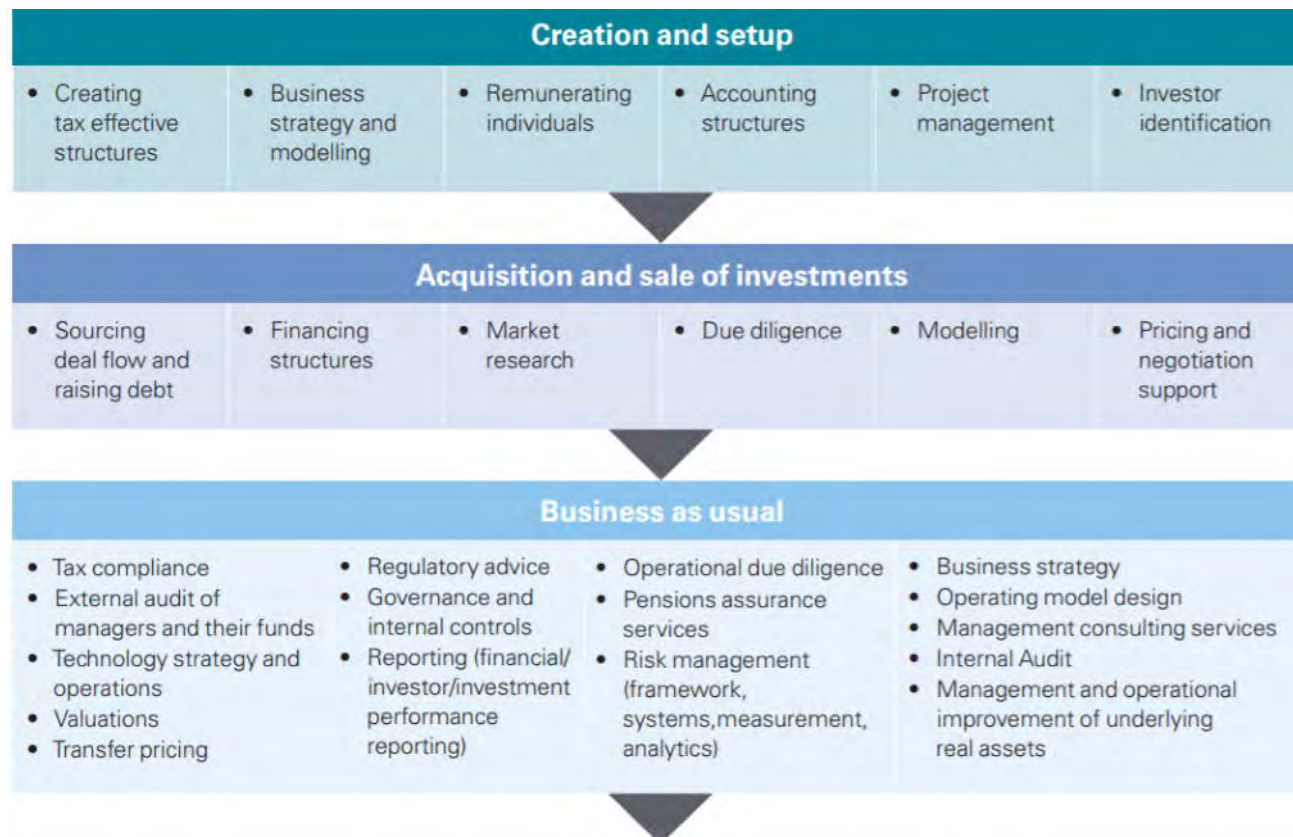
2.1. What do we mean if we say “Big Four?”

- Four companies that are among the largest financial and consulting service providers in the accounting and investment management industry are collectively referred as the “Big Four”, help anticipate what ‘s after what ‘s next:



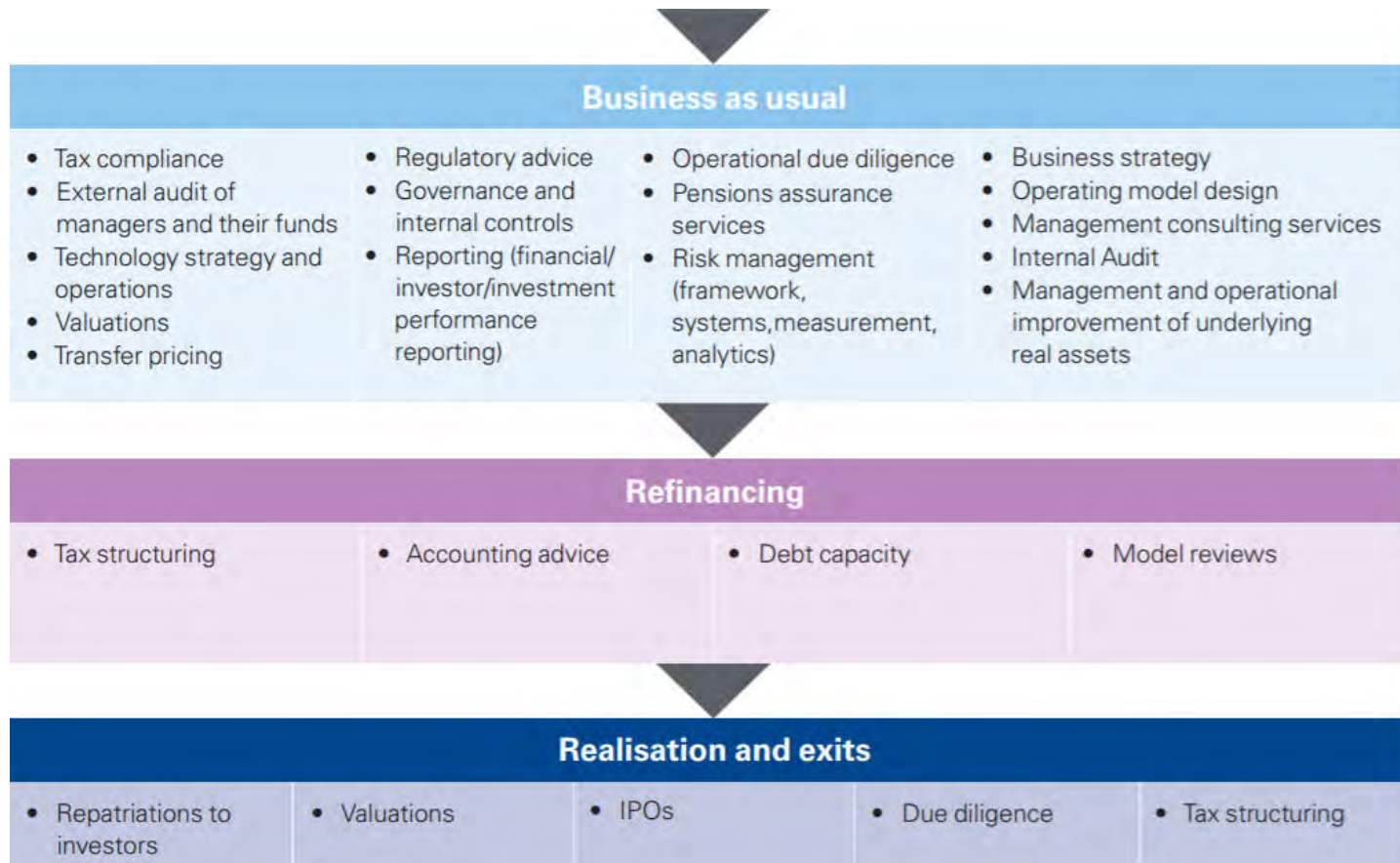


2.1.1 Example of global management services in KPMG (1)





2.1.1 Example of global management services in KPMG (2)





How digitization affect the alternative investment industry? (1)

The evolution of information technology:

- 1960s: the arrival of semiconductor-based microprocessors creating digital data
- 1970s: mainframe computers, capable of processing larger data in one location
- 1980s: distribution of data via remotely connected mini computers, the arrival of personal computers
- 1990s: the evolution continued to encompass mobile phones
- 2000s: the rise of internet and the evolution of smart phones permitting connectivity
- **5G** (fifth generation, digital tools having impact on business, markets, the economy and society)

“The latest phase of digital evolution is nowadays encompassing revolutionary technologies like blockchain, machine learning and robotic process automation. Speed, connectivity, insights, transparency and disintermediation are their hallmarks that are reshaping business models in finance.” (KPMG, 2018)





How digitization affect the alternative investment industry? (2)

“What the internet did to the music business, digitization will do to finance. It’s not about ‘if’ but ‘when’. (KPMG, 2018)

“In our due diligence, we use scraping analytics that extract data from websites, social media and prediction markets”. (KPMG, 2018)

The global survey of KPMG (2018) that covers 19 countries with **alternative assets under management** (a so-called **alternative AUM** or simply “**alternatives**”) of US\$2.6 trillion shows that:

A. Two **key segments in the alternative investment industry most amenable to digitization** are:

- **hedge funds**
- **private equity**

“One of the world’s largest hedge funds is now letting computers trade completely on their own.” (KPMG, 2018)





How digitization affect the alternative investment industry? (3)

- B. Alternatives face disruption, as digitization becomes the North Star of the industry
 - For example significant information advantages via machine learning
- C. Adoption of digital innovations so far has been a matter of more haste, less speed
 - For example delivering investible information and insights with the science of analytics:
 - descriptive analytics: what happened?
 - diagnostic analytics: why did it happen?
 - predictive analytics: what will happen?
 - prescriptive analytics: how do we react?
- D. Alternative investing will shift up a gear, as organic growth becomes the new mantra
 - in business models, will be **shift from product centricity to client centricity** as the alternative investment industry enters the third phase, AI 3.0.
- E. Business transformation requires digital leaders who can rewrite the traditional narrative on value creation





How digitization affect the alternative investment industry? (4)

Eight key digital innovations in investment industry, based on (KPMG, 2018)

1. Robotic process automation
2. Application programming interfaces
3. Big data
4. Social media
5. New digital platforms for next-gen investors
6. Cognitive computing and machine learning
7. Robo advisors
8. Blockchain

(KPMG, 2018)

“To succeed with tomorrow’s consumers, financial services must become a data-driven, differentiated and hyper-personalized experience.” (EY, 2019)





How digitization affect the alternative investment industry? (5)

“Digitization is primarily a business model innovation.” (KPMG, 2018)

Prominent pioneers like the following three tech giants:

- Amazon,
- Google
- and Uber

, have already redefined their industries by

- rewiring customer experience,
 - addressing unmet needs and creating new needs,
 - or finding cheaper ways of meeting existing needs in both B2B as well as B2C segments.
- (KPMG, 2018)

B2B (Business-to-Business) means trade between businesses rather than to consumers

B2C (Business-to-Consumer) means directly selling to consumers who are end-users





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Examples of activities in investment value chain (1)

1. Front office

- Portfolio risk management
- Research and securities selection
- Big data analytics
- Due diligence
- Algorithmic trading
- Product development and fund governance
- Crowdfunding
- Alpha generation

(KPMG, 2018)



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Examples of activities in investment value chain (2)

2. Middle office

- Risk and compliance
- Marketing and branding
- Client on-boarding
- Asset gathering via digital platforms for end-clients
- CRM that works in a cheaper, faster and better way
- Intermediary relationships
- Smart contracts

(KPMG, 2018)



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Examples of activities in investment value chain (3)

3. Back office

- Fund accounting
- Trade and settlements
- Transfer agent
- Valuations of illiquid assets
- Global custody
- Depository

(KPMG, 2018)



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3. Quantitative investment analysis: *Portfolio performance measurement*



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3.1 How to evaluate the performance of the portfolio (1)?

- The large amounts of money is being managed for other people (investors) by

Institutional money managers

Pension fund managers

Mutual fund managers



- Are they doing a beneficial job?

In other words, has actively managed fund better performance than a passively managed portfolio?

In order to answer, we need to evaluate the portfolio performance.

(CFA Program Curriculum, 2018)





3.1 How to evaluate the performance of the portfolio (2)?

- Performance has two main components:

Risk

Return



, while **return maximization is a common objective**, comparing just **return** of a portfolio with that of benchmark (market, index) **is not sufficient**.

- Risk averse investors require **compensation for higher risk** in the form of higher expected returns.
- Based on finance theory in the long run, investors should be **compensated with additional mean return at least above the risk-free rate** for bearing additional **risk**, if the risky portfolio is well diversified.





3.1 How to evaluate the performance of the portfolio (3)?

- Four ratios are commonly used in performance evaluation

Sharpe Ratio

Treynor Ratio

M-Squared (M^2)

Jensen's Alpha

, and are considered as important **risk-adjusted performance measures**.

(CFA Program Curriculum, 2018)





3.1.1 Sharpe Ratio

- $Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p}$
- Is also called **reward-to-variability ratio**
- The ratio uses the **total risk of the portfolio**, not its systematic risk

Was created by William Sharpe (Sharpe, 1964)
where R_p = the mean return to the portfolio,
 R_f = the mean return to a risk-free asset,
 σ_p = the standard deviation of return on the portfolio

How to interpret *Sharpe ratio*?

- The **portfolio with the highest Sharpe Ratio has the best performance**
- The **portfolio with the lowest Sharpe Ratio has the worst performance**
- Warning: numerator should be positive to give meaningful results

Two limitations

- Using the total risk (standard deviation of returns) when only systematic risk is priced
- **The ratio itself is not informative (eg 0.2), only if compared to Sharpe ratio of another portfolio**

(CFA Program Curriculum, 2018)





3.1.1.1 Calculation of the Sharpe Ratio (1)

- Example 1

Consider the performance of two exchange traded funds. SPDR S&P 500 seeks to track the investment results of the S&P 500 Index (large capitalization US stocks) and iShares Russell 2000 Index seeks to track the investment results of the Russell 2000 Index (small capitalization US stocks). Table below presents the historical arithmetic mean return, along with the historical standard deviation of returns, for annual returns series of these two funds and the US 30-day T-bill during the 2003–2012 period.

Fund/T-Bill	Arithmetic Mean (%)	Standard Deviation of Return (%)
iShares Russell 2000 Index	9.26	22.36
SPDR S&P 500 Index	6.77	19.99
30-day T-bill	1.58	1.78

Sources: finance.yahoo.com and www.federalreserve.gov.





3.1.1.1 Calculation of the Sharpe Ratio (1)

- Example 1

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

$$\text{Sharpe Ratio } iShares \text{ Russell } 2000 = \frac{9.26 - 1.58}{22.36} = 0.34$$

$$\text{Sharpe Ratio } SPDR \text{ S\&P } 500 = \frac{6.77 - 1.58}{19.99} = 0.26$$

Although US small stocks (iShares Russell 2000 Index) had a higher standard deviation, they performed better than the US large stocks (SPDR S&P 500 Index), as measured by the Sharpe ratio





Calculation of Arithmetic Mean

- Example 2

During the year, profit as a percentage of revenue for BJ's Wholesale club, Costco Wholesale Corporation, and Wal-Mart Stores was 0.9 percent, 1.6 percent, and 3.5 percent, respectively, according to the Fortune 500 list for 2012. Calculate the mean.

The arithmetic mean is the sum of the observations divided by the number of observations.

$$\mu = \frac{\sum_{i=1}^N X_i}{N}$$

where N = the number of observations

X_i = the i th observation.

Mean profit as a % of revenue was $\mu = (0.9 + 1.6 + 3.5)/3 = 6/3 = 2 \%$

The arithmetic mean profit as a percentage of revenue was 2 percent.





Calculation of Standard Deviation (1)

- Example 3

The total returns for two mutual funds in the period between 2008–2012 were:

Year	Selected American Shares (SLASX)	T. Rowe Price Equity Income (PRFDX)
2008	–39.44%	–35.75%
2009	31.64	25.62
2010	12.53	15.15
2011	–4.35	–0.72
2012	12.82	17.25

Source: performance.morningstar.com.

Find the standard deviation for both mutual funds.

The standard deviation as a positive square root of variance:

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (X_i - \mu)^2}{N}}$$



where μ = the population mean
N = the size of population

(CFA Program Curriculum, 2018)



Calculation of Standard Deviation (2)

- Example 3 (cont.1)

For SLASX

The sample mean is

$$R = (-39.44 + 31.64 + 12.53 - 4.35 + 12.82) / 5 = 13.20 / 5 = 2.64\%$$

The squared deviations from the mean are

$$(-39.44 - 2.64)^2 = (-42.08)^2 = 1,770.73$$

$$(31.64 - 2.64)^2 = (29.00)^2 = 841.00$$

$$(12.53 - 2.64)^2 = (9.89)^2 = 97.81$$

$$(-4.35 - 2.64)^2 = (-6.99)^2 = 48.86$$

$$(12.82 - 2.64)^2 = (10.18)^2 = 103.63$$

. The sum of the squared deviations from the mean is

$$1,770.73 + 841.00 + 97.81 + 48.86 + 103.63 = 2,862.03.$$

Divide the sum of the squared deviations from the mean by $n - 1$:

$$2,862.03 / (5 - 1) = 2,862.03 / 4 = 715.51$$

To find the standard deviation, we take the positive square root of variance

$$\sigma = \sqrt{715.51} = 26.7\%$$

The standard deviation for SLASX is 26.7%

(CFA Program Curriculum, 2018)





Calculation of Standard Deviation (3)

- Example 3 (cont.2)

For PRFDX

The sample mean is

$$R = (-35.75 + 25.62 + 15.15 - 0.72 + 17.25)/5 = 21.55/5 = 4.31\%$$

The squared deviations from the mean are

$$(-35.75 - 4.31)^2 = (-40.06)^2 = 1,604.80$$

$$(25.62 - 4.31)^2 = (21.31)^2 = 454.12$$

$$(15.15 - 4.31)^2 = (10.84)^2 = 117.51$$

$$(-0.72 - 4.31)^2 = (-5.03)^2 = 25.30$$

$$(17.25 - 4.31)^2 = (12.94)^2 = 167.44$$

. The sum of the squared deviations from the mean is

$$1,604.80 + 454.12 + 117.51 + 25.30 + 167.44 = 2,369.17$$

Divide the sum of the squared deviations from the mean by $n - 1$:

$$2,369.17/4 = 592.29$$

To find the standard deviation, we take the positive square root of variance

$$\sigma = \sqrt{592.29} = 24.3\%$$

The standard deviation for PRFDX is 24.3%

(CFA Program Curriculum, 2018)





3.1.2 Treynor Ratio

- $Treynor\ Ratio = \frac{R_p - R_f}{\beta_p}$

was created by Jack Treynor (Treynor, 1961, 1962)
where R_p = the mean return to the portfolio,
 R_f = the mean return to a risk-free asset,
 β_p = beta risk

- is a simple extension of the Sharpe ratio and resolves its limitation on total risk
- The ratio substitutes **beta risk** ie systematic risk, for total risk and measures a portfolio's excess returns to the portfolio's beta.

How to interpret *Treynor ratio*?

- The **portfolio with the highest Treynor Ratio has the best performance**
- The **portfolio with the lowest Treynor Ratio has the worst performance**
- Warning: numerator and denominator should be positive to give meaningful results

Limitation The ratio itself is not informative (eg 0.2), only if compared of another portfolio

Similarly like Sharpe ratio we do not whether either of the compared compared is better than the passive portfolio portfolios -> Jensen's alpha and M² attempt to resolve this problem

(CFA Program Curriculum, 2018)





3.1.3 M-Squared (M^2)

was created by Franco Modigliani and his granddaughter Leah Modigliani -> therefore: M^2

where R_p = the mean return to the portfolio,

R_f = the mean return to a risk-free asset,

σ_m = the standard deviation of return on the market

σ_p = the standard deviation of return on the portfolio

$$M^2 = (R_p - R_f) * \frac{\sigma_m}{\sigma_p} - (R_m - R_f)$$

- gives rankings identical to Sharpe ratio, but in percentage terms easier to interpret and is directly compared with the market return (benchmark)
- By using M^2 , we are able to determine the rank of a portfolio and, if any, of our portfolios beat the market on a risk-adjusted basis.

How to interpret M^2 ?

- A portfolio that matches the performance of the market will have an M^2 of zero
- A portfolio that outperforms the market will have an M^2 that is positive.

Limitation

- is similarly like Sharpe ratio, based on total risk, not beta risk.

(CFA Program Curriculum, 2018)





3.1.4 Jensen's Alpha

where R_p = the actual portfolio return

R_f = the average risk-free rate

β_p = beta risk

α_m of the market is zero

- $\alpha_p = R_p - [R_f + \beta_p * (R_m - R_f)]$
- is a simple extension of the Sharpe ratio and resolves its limitation on total risk using **the beta of the portfolio β_p and the CAPM** to quantify the risk-adjusted return of the portfolio (it substitutes **beta risk β_p** ie systematic risk, for total risk, like Treynor ratio)
- the systematic risk **β_p** can be measured by estimating the market model, by **regressing the portfolio's daily return on the market's daily return**
- Jensen 's alpha **α_p** is simply a measure of the portfolio's performance relative to the market portfolio, measured as the difference between the actual portfolio return and the calculated risk-adjusted return

How to interpret *Jensen 's Alpha ratio*?

- If **α_p is positive, then the portfolio has outperformed the market**
- If **α_p is negative, then the portfolio has underperformed the market**
- Jensen's alpha is the maximum amount that we should be willing to pay the manager to manage your money. (CFA Program Curriculum, 2018)





Interpreting Jensen's Alpha ratio

- Example 4 **Estimated portfolio's (A) alpha is 2 percent and another portfolio's (B) alpha is 5 percent. Quantify which portfolio outperformed and interpret the results to the investor.**

$$\alpha_{p(A)} = R_p - [R_f + \beta_p * (R_m - R_f)] = 2 \%$$

$$\alpha_{p(B)} = R_p - [R_f + \beta_p * (R_m - R_f)] = 5 \%$$

by the definition: $\alpha_m = 0 \%$; $\beta_m = 1$;

-> portfolio A outperformed the market by 3 %

-> portfolio B outperformed the market by 5 %

$$\alpha_{p(A)} < \alpha_{p(B)} \rightarrow 5 \% - 2 \% = 3 \%$$

The portfolio B has outperformed the portfolio A by 3 percentage points and the market by 5 percentage points. Jensen's alpha is the maximum amount that the investor should be willing to pay the manager to manage his/her money.





3.2 Determining and estimating beta (1)

Asset (portfolio) Beta

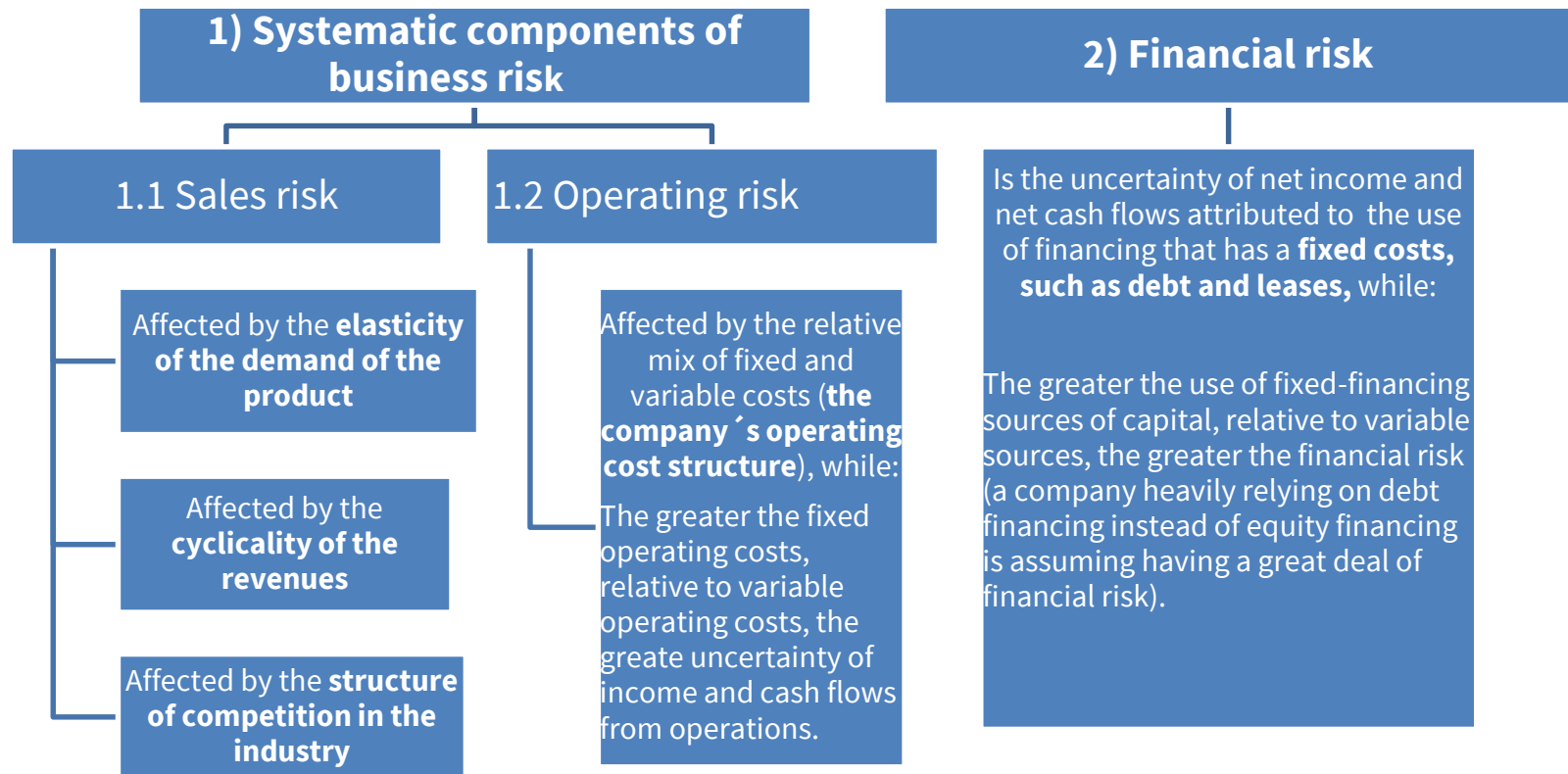
- denoted by the Greek letter beta (β) is frequently used finance risk concept in investment practice, based on standard deviation (volatility) and simply reflects the business risk of the assets referred commonly as **the asset's systematic risk**
- by the definition **a measure of the sensitivity of a given investment (asset) or portfolio to movements in the overall market**
- we can meet beta for example in 1) **CAPM model**, it means in the equation describing the expected return on any asset (or portfolio), which is a linear function of its β relative to the market portfolio, in 2) **multifactor models**, or 3) or when estimating **Treynor and several other risk adjusted performance ratios**.
- The beta of a company or project is affected by the **systematic components of business risk** and by **financial risk** (see next slide for more details).
- Simply $\beta > 1$ is **more risky (volatile) than the market** and $\beta < 1$ is **less risky (volatile than the market)**
- The **high-beta stocks** are considered as high-risk investments usually included in the portfolio to reap short-term rewards and require active trading.





3.2 Determining and estimating beta (2)

The **beta** of a company or project is affected by **2 components**:





3.2 Determining and estimating beta (2)

Asset (portfolio) beta enters commonly into

1) Single-factor approach (CAPM)

- if using the **CAPM** to estimate the cost of equity (r_e , $E(R_i)$) for valuation purposes, typically in practice the β is being estimated relative to an equity market index. In that case, the market premium estimate $[E(R_m) - R_f]$ we are using is actually an estimate of the equity risk premium (ERP).

$$E(R_i) = R_f + \beta_i * [E(R_m) - R_f]$$

2) Multi-factor approach

- Is an alternative to CAPM and is saying risks can not be captured only by the market index alone (eg S&P 500) but with various other factors that are sources of priced risks (risk for which investors demand compensation for bearing), including macroeconomic factors and company-specific factors.

$$E(R_i) = R_f + \beta_{i1} * [F_1] + \beta_{i2} * [F_2] + \dots + \beta_{ij} * [F_j]$$

β_{ij} is the stock_i's sensitivity to changes in the j^{th} factor

3) Several risk adjusted performance ratios





3.2 Determining and estimating beta (3)

1. Single-factor approach (CAPM)

$$E(R_i) = R_f + \beta_i * [E(R_m) - R_f]$$

, where $E(R_i)$ is the sum of the risk-free rate (R_f) of interest and a premium for beat the stock 's market risk

β_i = the return sensitivity of stock i to changes in the market return

$E(R_m)$ = the expected return on the market

$[E(R_m) - R_f]$ is the expected market risk premium (ERP)

A common proxy for the R_f is the yield on a default-free government debt instrument (eg 10-year Treasury bond when evaluating 10-year project)

- the CAPM approach is the one of commonly used approaches (others eg dividend discount model, or the bond yield plus risk premium method) to estimate the cost of equity (r_e or $E(R_i)$) to inform about rate of return required by a company's common shareholders
- Limitation: the estimation of the r_e is challenging because of the uncertain nature of the future cash flows in terms of the amount and timing.





Beta and Capital Asset Pricing Model (CAPM)

- Example 5

Alpha Industries wants to know its cost of equity. Its CFO believes the risk-free rate is 5 percent, equity risk premium is 7 percent, and Valence's equity beta is 1.5. What is Alpha's cost of equity using the CAPM approach?

Solution

$$E(R_i) = R_f + \beta_i * [E(R_m) - R_f]$$

Cost of common stock = 5 % + 1.5 * (7 %) = 15.5 %

7 percent is the expected market premium which means that investors demand 7 percent for investing in a market portfolio (eg index fund tracking whole stock market) relative to the risk free rate. Based on CAPM model shareholders should require at least 15.5 % as a rate of return.





3.2 Determining and estimating beta (4), ERP

The equity risk premium (ERP: $(R_m - R_f)$)

- is the **expected return on equities minus the risk-free rate**; **the premium that investors demand for investing in equities.**

To estimate ERP can be commonly used

1. Historical equity risk premium approach

- the estimate of a country's equity risk premium that is based upon the historical averages of the risk-free rate and the rate of return on the market portfolio.
- *For example finding the annualized US equity (using the historical returns to the S&P 500 Index to estimate risk premium for US equities) ERP relative to US Treasury Bills (or bonds). For more, see eg. Dimson, Marsh and Staunton (2003).*

2. Survey approach

- based upon estimates of ERP provided by a panel of finance experts.

3. Dividend discount model based approach

- the market rate of return is estimated as the sum of the dividend yield and the growth rate in dividends for a market index. Subtracting the risk-free rate of return from the estimated market return produces an estimate for the equity risk premium.



The historical premium approach illustration example

- Example 6

Suppose that the arithmetic average T-bond rate observed over the last 100 years is an unbiased estimator for the risk-free rate and amounts to 5.4 percent. Likewise, suppose the arithmetic average of return on the market observed over the last 100 years is an unbiased estimator for the expected return for the market. The average rate of return of the market was 9.3 percent. Calculate the equity risk premium. Describe also the following steps to calculate the cost of equity by using the CAPM.

Solution

$$E(R_i) = R_f + \beta_i * [E(R_m) - R_f]$$

$$ERP = [E(R_m) - R_f] = 9.3 \% - 5.4 \% = 3.9 \%$$

The equity risk premium is 3.9 percent. Once we have an estimate of the equity risk premium, we can adjust for the specific systematic risk of the particular company or project. We adjust for the specific systematic risk by multiplying the market risk premium by beta to arrive at the company's or project's risk premium, which we then add to the risk-free rate to determine the cost of equity within the framework of the CAPM.





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3.2 Determining and estimating beta (5)

Common methods of estimation the asset's beta (1)

1. Market model regression
2. Pure-play method





3.2 Determining and estimating beta (5)

Common methods of estimation the asset's beta (2)

1. Market model regression

- Using market model regression of the company's stock R_i against market returns R_m over T periods (for more see the equation developer Jensen, 1969)

$$R_{it} = \hat{a} + \hat{b} * R_{mt} \quad t = 1, 2, \dots, T$$

where \hat{a} is the estimated intercept

\hat{b} is the estimated slope of the regression that is used as an estimate of beta

- See CFA Program Curriculum (2018) for recommended additional adjustments
- For **publicly traded companies**, the beta can be sourced from several web sites like Reuters or beta can be estimated using **simple regression**, while the stock return data are commonly available.





3.2 Determining and estimating beta (5)

Common methods of estimation the asset's beta (3)

The challenge is to estimate beta for a **not publicly traded company** or not a typical (average) project, **in these cases proxying for the beta**.

2. Pure-play method (1)

- β can be alternatively estimated by this method, for a company or project, **by using a comparable company's beta and adjusting it for financial leverage differences**.
- **A comparable company is the company having similar business risk**, in the same industry that is in that single line of business.
- **The analyst must adjust for differing degrees of financial leverage**, requiring *unlevering* and *levering* the beta, while the **beta of the comparable is firstly unlevered by removing the effects of its financial leverage**.” (the developer of the process of unlevering and levering beta Hamada (1972), based on the capital structure theories of Franco Modigliani and Merton Miller).
- **The unlevered beta** is commonly referred as **the asset beta** because **it reflects the business risk of the assets**.





3.2 Determining and estimating beta (5)

2. Pure-play method (2): how to determine the unlevered beta (1)

- we must determine the relationship between a company's asset beta and its equity beta in order to unlever its equity beta to estimate its asset beta (company's risk).
- The company's risk is shared between creditors and owners, considering the effects of the tax-deductibility of interest related to debt financing:

$$\beta_{asset} = \beta_{debt} * w_d + \beta_{equity} * w_e$$

$$= \beta_{debt} * \left(\frac{(1-t)*D}{(1-t)*D+E} \right) + \beta_{equity} * \left(\frac{E}{(1-t)*D+E} \right)$$

where β_{debt} = the weighted average of the company's creditors' market risk

β_{equity} = the weighted average of market risk of the owners

E = market value of equity

D = market value of debt

w_d = proportion of debt

w_e = proportion of equity

t = the marginal tax rate

- Considering a general assumption a company's debt is without market risk, while the returns on debt do not vary with the returns on the market:

$$\beta_{asset} = \beta_{equity} w_e = \beta_{equity} * \left(\frac{E}{(1-t)*D+E} \right) = \beta_{equity} * \left[\frac{1}{1 + \left((1-t) * \frac{D}{E} \right)} \right]$$





3.2 Determining and estimating beta (5)

2. Pure-play method (3): Steps to estimate beta

- Step 1: **Select the comparable** Determine comparable company or companies. These are companies with similar business risk.
- Step 2: **Estimate comparable's beta** Estimate the equity beta of the comparable company or companies.
- Step 3: **Unlever the comparable's beta** Unlever the beta of the comparable company or companies, removing the financial risk component of the equity beta, leaving the business risk component of the beta.
- Step 4: **Lever the beta for the project's financial risk** Lever the beta of the project by adjusting the asset beta for the financial risk of the project.





3.2 Determining and estimating beta (5)

2. Pure-play method (2): how to determine the unlevered beta (2)

- Estimating the asset (unlevered) beta for the comparable:

$$\beta_{U,comparable} = \left[\frac{\beta_{L,comparable}}{1 + \left((1 - t_{comparable}) * \frac{D_{comparable}}{E_{comparable}} \right)} \right]$$

,where $\beta_{L,comparable}$ is known

- Calculating the equity risk of the project (asset), considering its financial leverage (levered beta of the project (asset):

$$\beta_{L,project} = \beta_{U,comparable} * \left[1 + \left((1 - t_{project}) * \frac{D_{project}}{E_{project}} \right) \right]$$





Estimating beta by using pure-play method illustration examples (1)

■ Example 7

We want to evaluate a project that will be financed with debt and equity in a ratio of 0.4:1 [a debt-to-equity ratio of 0.4, corresponding to approximately $0.4/(0.4 + 1.0) = €0.286$ for each euro of capital needed]. We find a comparable company operating in the same line of business as the project. The marginal tax rate for the company sponsoring the project and the comparable company is 35 percent. The comparable company has a beta of 1.2 and a debt-to-equity ratio of 0.125. Find the levered beta for the project which enters further into CAPM and cost of capital (WACC) estimates.

Solution

$$\beta_{U,comparable} = \left[\frac{1.2}{1 + ((1 - 0.35) * 0.125)} \right] = 1.1098$$

$$\beta_{L,project} = 1.1098 [1 + ((1 - 0.35) * 0.4)] = 1.3983$$

The unlevered beta of the comparable is 1.1098. The levered beta for the project is 1.3983 and this beta enters into CAPM, etc. estimates.

(In this example, the weights are $w_d = 0.4/1.4 = 0.2857$ and $w_e = 1/1.4 = 0.7143$)





Estimating beta by using pure-play method illustration examples (2)

■ Example 8

Raymond Cordier is the business development manager of Aerotechnique S.A., a private Belgian subcontractor of aerospace parts, not listed on the Belgian stock exchange. Mr. Cordier needs to evaluate the levered beta for the company. 1) He has access to the following information. The average levered and average unlevered betas for the group of comparable companies operating in different European countries are 1.6 and 1.0, respectively. Aerotechnique's debt-to-equity ratio, based on market values, is 1.4. and its corporate tax rate is 34 percent. 2) How would the estimation of asset beta change if the company would be publicly traded and the company's stock beta would be 1.3, with that the market value of equity and debt respectively, C\$540 million and C\$720 million. Assume the marginal tax rate of this company 40 percent.

Solution

$$1) \beta_{Aerotechnique} = 1.0 * [1 + ((1 - 0.34) * 1.4)] = 1.924$$

$$2) \beta_{Aerotechnique} = \left[\frac{1.3}{1 + ((1 - 0.40) * \frac{720}{540})} \right] = 0.72$$

The estimated beta for Aerotechnique is in the case 1) 1.92 and in the case 2) 0.72.



3.3 Portfolio performance evaluation concluding example

■ Example 9 (1)

A pension fund (A) has employed three investment managers (information about the managers is given in the below table), each of whom is responsible for investing in one-third of all asset classes so that the pension fund has a well-diversified portfolio. Calculate 1) the expected return, Sharpe ratio, Treynor ratio, M2, and Jensen's alpha, 2) analyze your results and 3) plot the returns and betas of these portfolios.

Manager	Return	σ	β
X	10%	20%	1.1
Y	11	10	0.7
Z	12	25	0.6
Market (M)	9	19	
Risk-free rate (R_f)	3		



Show calculation only for manager X and for homework practice it for manager Y and Z.





3.2 Portfolio performance evaluation concluding example

▪ Example 9 (2)

Calculations for manager X:

1) Expected return: $E(R_x) = R_f + \beta_X * [E(R_m) - R_f] = 0.03 + 1.10 * (0.09 - 0.03) = 0.096 = 9.6\%$

Sharpe ratio = $\frac{R_X - R_f}{\sigma_X} = \frac{0.10 - 0.03}{0.20} = 0.35$

Treynor ratio = $\frac{R_X - R_f}{\beta_X} = \frac{0.10 - 0.03}{1.1} = 0.064$

$M^2 = (R_X - R_f) * \frac{\sigma_m}{\sigma_X} - (R_m - R_f) = (0.10 - 0.03) * \frac{0.19}{0.20} - (0.09 - 0.03) = 0.0065 = 0.65\%$

$\alpha_X = R_X - [R_f + \beta_X * (R_m - R_f)] = 0.10 - [0.03 + 1.1 * 0.06] = 0.004 = 0.40\%$

Manager	R_X	σ_X	β_X	$E(R_x)$	Sharpe Ratio	Treynor Ratio	M^2	α_X
X	10.0 %	20.0 %	1.10	9.6 %	0.35	0.064	0.65 %	0.40 %





3.2 Portfolio performance evaluation concluding example

▪ Example 9 (3)

Manager	R_X	σ_X	β_X	$E(R_X)$	Sharpe Ratio	Treynor Ratio	M^2	α_X
X	10.0 %	20.0 %	1.10	9.6 %	0.35	0.064	0.65 %	0.40 %

2) Portfolio X along the SML

; by the definition from finance theory*:

$$\beta_m = 1;$$

$$\alpha_m = 0 \%$$

σ and β of R_f are both zero

(* see more calculations on another slaid)

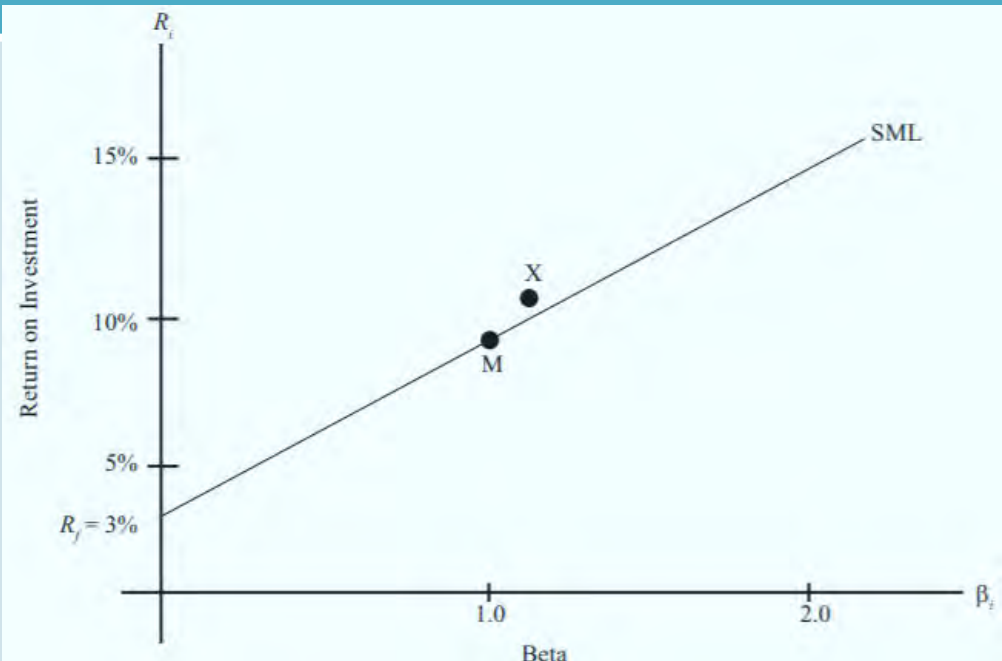


Exhibit confirms the manager X outperformed the benchmark because the portfolio X lies above the SML.



3.2 Portfolio performance evaluation concluding example

▪ Example 9 (4 homework solution)

(CFA Program Curriculum, 2018)

Manager	R_i	σ_i	β_i	$E(R_i)$	Sharpe Ratio	Treynor Ratio	M^2	α_i
X	10.0 %	20.0 %	1.10	9.6 %	0.35	0.064	0.65 %	0.40 %
Y	11.0 %	10.0 %	0.70	7.2 %	0.80	0.114	9.20 %	3.80 %
Z	12.0 %	25.0 %	0.60	6.6 %	0.36	0.150	0.84 %	5.40 %
M	9.0 %	19.0 %	1.00	9.0 %	0.32	0.060	0.00 %	0.00 %
R_f	3.0 %	0.0 %	0.00	3.0 %	—	—	—	0.00 %

- 1) For R_f (risk-free rate) **ratios** can not be calculated because the risk-free asset has zero risk (beta = 0) and α_f is zero.
- 2) M (Market portfolio, benchmark): positive absolute measures of performance, Sharpe and the Treynor ratio inform us the market portfolio earns a return that excess of that of risk-free rate. M^2 and α_i are performance measures relative to market, therefore both equal to zero for the M.





3.2 Portfolio performance evaluation concluding example

▪ Example 9 (5, homework solution)

(CFA Program Curriculum, 2018)

Manager	R_i	σ_i	β_i	$E(R_i)$	Sharpe Ratio	Treynor Ratio	M^2	α_i
X	10.0 %	20.0 %	1.10	9.6 %	0.35	0.064	0.65 %	0.40 %
Y	11.0 %	10.0 %	0.70	7.2 %	0.80	0.114	9.20 %	3.80 %
Z	12.0 %	25.0 %	0.60	6.6 %	0.36	0.150	0.84 %	5.40 %
M	9.0 %	19.0 %	1.00	9.0 %	0.32	0.060	0.00 %	0.00 %
R_f	3.0 %	0.0 %	0.00	3.0 %	—	—	—	0.00 %

3) All managers X, Y, Z have Sharpe and Treynor ratios > those of the market, satisfying the fund managers with their performance. Manager X has the worst performance, irrespective of whether total risk or systematic risk is considered for measuring performance. (See another slide for more interpretation results based on relative rankings).





3.2 Portfolio performance evaluation concluding example

- Example 9 (6, homework solution)

(CFA Program Curriculum, 2018)

Rank	Sharpe Ratio	Treynor Ratio	M^2	α_i
1	Y	Z	Y	Z
2	Z	Y	Z	Y
3	X	X	X	X
4	M	M	M	M
5	–	–	–	R_f

4) Comparing Y and Z, Y performs much better than Z when total risk is considered. Contrary, when systematic risk is used, Z outperforms Y, indicating that Z has done a better job of generating excess return relative to systematic risk than Y.



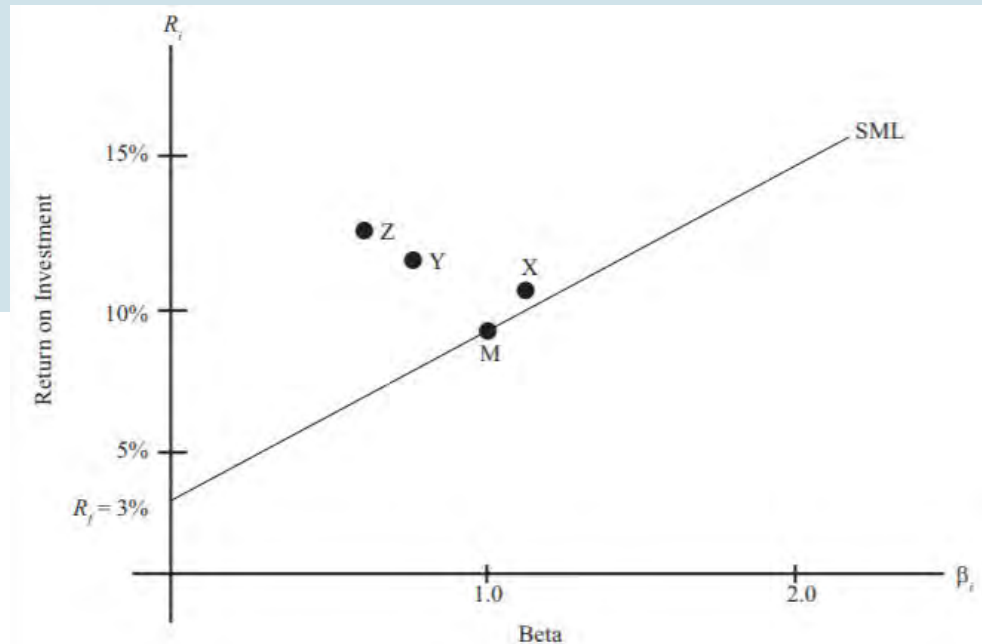


3.2 Portfolio performance evaluation concluding example

- Example 9 (7, homework solution)

(CFA Program Curriculum, 2018)

4) We can see that all managers outperform the benchmark because all three points are lying above the SML line.





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Capital Markets

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Essentials of Quantitative Investment Analysis for Qualified
Investment Professional

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2020

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INTRODUCTION

Principles of quantitative investment analysis are widely applied in the professional investment practice and are considered as essential tools to understand the quantitative aspects of investing.

This handbook heavily builds up on CFA Institute Investment Series books oriented on quantitative methods and their application in investment processes.

Various books (see e.g., Defusco et al., 2007; Bodie et al., 2013; Defusco et al., 2015a; Defusco et al., 2015b; CFA program curriculum, 2017, 2018; Šoba and Širůček, 2017; CFA Institute, 2019) attempt to describe and explain a field of international business environment and finance from different angles of view, however, all of these books ensure sharpening the skills by numerous practice problems and example-driven coverage suitable for both, a novice investor or an experienced practitioner.

Among the broadly discussed topics in the quantitative investment analysis-based books belong namely the time value of money, discounted cash flow applications, statistical concepts and market returns, probability concepts and common probability distributions, sampling and estimation, hypothesis testing, correlation and regression, multiple regression, time-series analysis, and multifactor models.

The main goal of this handbook is to provide the participants of the summer school course in limited number of pages the general theoretical background, practical examples, easy to follow steps and even with an inspiration on a journey of understanding of **essentials of quantitative investment analysis**, in particular is due to the limited space dedicated to b





both first two aspects- **the time value of money**, and **discount cash flow applications**. This handbook is limited only to these fields, while for learning other aspects of quantitative investment analysis is recommended to read aforementioned literature.

This handbook is structured as follows. It focuses on aspects of the time value of money and discount cash flow applications and is heavily oriented on practical applications with lot of examples and cases.



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1 INVESTMENT MANAGEMENT INDUSTRY AND QUALIFIED INVESTMENT PROFESSIONAL: GENERALLY ACCEPTED MAINSTREAM AND FRONTIER CONCEPT

Various definitions of both terms, either investment management industry, and qualified investment professional can be found in the investment knowledge- based literature.

Based on one of the biggest auditing companies, KPMG (2014) **the investment management industry** is referred as a vibrant market place where the globe for sources of wealth and attractive opportunities in traditional and alternative investments are being scoured by investors.

Based on U.S. self-regulatory organization, Financial Industry Regulatory Authority (FINRA, 2019), a **qualified investment professional** keep informal about how financial markets and economy are affecting investment portfolio, can assist with financial goal setting and can help to make investment decisions. Several types of investment professionals can be distinguished, including brokers, investment advisers and financial planners.

Example 1- The opportunities and challenges facing currently by the investment management industry

- steady recovery of the investment management industry since the 2008 financial crisis
- the value of assets under management (“AUM”) reaching almost 80 trillion USD
- regulatory change particularly in US and Europe
- switching from the survival in the short term to make their businesses sustainable growth, over the next 10 to 20 years
- core current discussions on megatrends (for example, demographics, technology or the environment) impacting business in the future.

(KPMG, 2014; Statista, 2017)



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1.1 Identifying and verifying must-have skills, knowledge and abilities of professional investor and practice analysis

Based on U.S. self-regulatory organization, Financial Industry Regulatory Authority (FINRA, 2019), a **qualified investment professional** keep informal about how financial markets and economy are affecting investment portfolio, can assist with financial goal setting and can help to make investment decisions. Several types of investment professionals can be distinguished, including brokers, investment advisers and financial planners.

The skills, knowledge and abilities professional investor and practice analyst need to have, it means grounded in investment management practice, are keeping verified by panels and surveys by CFA Institute, and synthesized to topics into a so-called Candidate Body of Knowledge (CBOK) used for creating CFA Curriculum and for CFA examination. CFA Institute is a global association of investment professionals.

These detected topics ensure remains aligned with what is required of today's—and tomorrow's—investment professionals. Example 2 shows the process of gathering topics for CBOK in CFA Institute.

Example 2- The process of identifying and verifying must-have skills, knowledge and abilities for investment management practice: Candidate Body Of Knowledge (CBOK) of CFA Institute



(Sourced from <https://www.cfainstitute.org/-/media/documents/factsheet/practice-analysis-fact-card.ashx>)



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CFA Institute gathers furthermore a so-called Global Body of Investment Knowledge (GBIC) representing a comprehensive outline of knowledge for the investment profession and includes actual mainstream and frontier concepts ensured by the Education Advisory Committee (EAC) of CFA Institute (regularly updated every five years from 1995).

Based on global investment association CFA Institute (2019), the current mainstream and frontier must-have learned concepts widely accepted as core investment knowledge cover appropriately Global Body of Investment Knowledge (GBIC®). Consequently being a comprehensive outline of knowledge for the investment profession, GBIC includes ten following fields:

- 1. Ethical and professional standards**
- 2. Quantitative methods**
- 3. Economics**
- 4. Financial reporting and analysis**
- 5. Corporate finance**
- 6. Equity investments**
- 7. Fixed income**
- 8. Derivatives**
- 9. Alternative investments**
- 10. Portfolio management and wealth planning**

Each field will be more elaborated into sub-topics within the following examples.

Example 3- GBIC®: Ethical and professional standards

- are one of the major knowledge areas needed to be learned to become a qualified investment professional or financial adviser.
- 3 main areas of ethical and professional standards covers:



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1. Applicable laws and regulations
 - a. *Governmental institutions*
 - b. *Securities and Exchange Commission and equivalent bodies in other countries*
 - c. *Rules and procedures regarding corporate transactions and insider trading*
2. Professional standards of practice
 - a. *The Code of Ethics*
 - b. *Standards of Professional Conduct*
3. Ethical practices and guidelines
 - a. *Corporate Government,*
 - b. *Soft Dollar Standards,*
 - c. *Fiduciary Duty,*
 - d. *Insider Trading,*
 - e. *Personal Investing,*
 - f. *Research objectivity Standards,*
 - g. *Trade Management Guidelines,*
 - h. *Analyst/ Corporate Relations Guidelines.*

(CFA Institute, 2019)

Example 4- GBIC®: Quantitative methods for quantitative investment analysis

- Time value of money
- Basic statistical and probability concepts
- Probability distributions
- Sampling and estimations
- Hypothesis testing
- Correlation analysis and regression
- Time series analysis
- Simulation analysis
- Scenario & sensitivity analysis
- Technical analysis

(CFA Institute, 2019)



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Example 5- GBIC®: Economics for investment decision makers

- Time value of money
- Basic statistical and probability concepts
- Probability distributions
- Sampling and estimations
- Hypothesis testing
- Correlation analysis and regression
- Time series analysis
- Simulation analysis
- Scenario & sensitivity analysis
- Technical analysis

(CFA Institute, 2019)

Example 6- GBIC®: Financial reporting and analysis, and international statement analysis

- Financial reporting system & Financial reporting quality
- Principal financial statements
- Analysis of inventories
- Analysis of long-lived assets
- Analysis of taxes
- Analysis of debt
- Analysis of off-balance-sheet assets and liabilities
- Analysis of pensions, stock compensation, and other employee benefits
- Analysis of inter-corporate investments
- Analysis of business combination
- Analysis of global operations, of financial instruments, derivatives and hedging activities
- Financial reporting in specialized industries and economic environments



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(CFA Institute, 2019)

Example 7- GBIC®: Equity investments and equity asset valuation

- Types of equity securities and their characteristics
- Equity markets: characteristics, institutions, and benchmarks
- Equity market valuation and return analysis
- Fundamental analysis (sector, industry, company) and the valuation of individual equity securities
- Special applications of fundamental analysis

(CFA Institute, 2019)

Example 8- GBIC®: Fixed income and fixed income analysis

- Fixed income securities and their characteristics
- Fixed income markets: characteristics, institutions and benchmarks
- Fixed income valuation and return analysis
- Term structure determination and yield spreads
- Analysis of interest rate risk
- Analysis of bond risk
- Valuing bonds with embedded options
- Structured products and valuing
- Trading strategies and their assessment
- -Influence of equity market changes on bond pricing
- Behavioral issues

(CFA Institute, 2019)

Example 9- GBIC®: Derivatives

- Types of derivative instruments and their characteristics
- Forward markets and instruments
- Futures markets and instruments
- Options markets and instruments



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- Swaps markets and instruments
- Credit derivatives markets and instruments
- Other derivatives issues

(CFA Institute, 2019)

Example 10- GBIC®: Alternative investments

- Types of alternative investment and their characteristics
- Real estate
- Investment companies
- Private equity
- Venture capital
- Hedge funds
- Closely-held companies and inactively traded securities
- Distressed securities/bankruptcies
- Commodities
- Tangible assets with low liquidity

(CFA Institute, 2019)

Example 11- GBIC®: Portfolio management and wealth planning

- Portfolio concepts from capital market theory
- Management of individual/family investor portfolios
- Management of institutional investor portfolios
- Pension plans and employee benefit funds
- Endowment funds and foundations
- Insurance companies
- Mutual funds, pooled funds, and ETFs
- Other institutional investors (banks, non-financial corporations)
- Investment manager selection
- Economic analysis and capital market expectations
- Asset allocation
- Portfolio construction and revision
- Equity portfolio management strategies
- Fixed income portfolio management strategies
- Alternative investment portfolio management strategies
- Risk management



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- Execution of portfolio decisions (trading)
- Performance measurement, attribution and appraisal
- Presentation of performance results

(CFA Institute, 2019)

Example 12- CFA Institute Research Challenge 2020– final round questions of judges examples

- I would like to hear risks that significantly can up or down in two years and can affect the recommendation. Discuss the main key scenarios.
- How will the debt develop in the future, the target debt net of 6.4% seems to be rather low.
- How was calculated the cost of debt.
- What other valuation methods could have been used based on the company profile.
- How was beta calculated. Using beta of sector seems to be not correct, and beta calculation 1.59 in terms of low debt and mature and profile company seems to be not correct. The CAPM model using S&P 500 model could have been used to estimate beta.
- Market vs. company risk, which was greater.
- Peer analysis meaning comparing main competitor of company should have been declared when applying financial analysis. Peer companies' multiples could have been shown.
- Could you comment risks regarding the business model of the company against other competitors.
- Before calculation FCFF model you have FCFF (deducted are interest expense), are they deducted, it is mistake, it is used for FCFE (free cash flow of equity) not in case of FCFF
- As you have buy recommendation which risks you see risks that can drive price down. You could have been elaborate one or two more.
- Slais could have been numbered.
- DCF (FCFF), the net debt is negative, because it has more short term investments than debt, and you should be adding not subtracting, so the value should be higher
- What are cost of equity and cost of debt assumptions.
- Do you know more about margins from clouds and compare them to main competitors?
/operating margin/ -> huge potential because of growth, and they compared to peers (cloud market share%)





- Specify more DCF model (why did you use FCFE and why did not use FCFF)
- Why you chose DCF model and what other valuation model you could have been used.
- Do you EVA model?
- Did you try to compare valuation based on segments?
- EBITDA why it will grow – economy of scale, higher margin of cloud, improvement of efficiencies, industry standards)
- Discuss whether there are signals for recession. For instance, by interpreting yield curves, etc.

Other notes from presentations

- Key markets-> Segments> CAGR rise, revenue forecast and related to market share of segments
- Which sector is the main in the segments they used in their business model, leading source of revenue
- Total sales based on markets and diversified across products and sectors

- Competitive environment
Swot analysis

Porter's five forces

Why will the market grow (cost efficiency, process improvements, etc competitive advantages

Competitive position of company in the market (market share, how many of Fortune 500 companies are using their products, distribution channels like linkedin), revenue growth and market share (two charts)

GDP/revenues -> is it cycle company?

- Financial analysis (eg we can call it Financial Stability)
Revenue forecast 2018-2025 (factors – markets growth /demand increasing and opportunities in a new market; firm – specific factors -)

FCF margin, high

High EBITDA margin

Lower capital

PE, PBV



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Peer comparison (P/E & EPS growth rate relationship in peers, eikon, forwarded 5-year eps growth rate divided by current P/E ratio), it is expensive (also eg dividend yields could have been used)

- Valuation (DCF,) vs Weighted average peer valuation
Revenue predictions & sensitivity analysis (decrease as a result of economic slowdown)
Tax rate – last %
Sales growth forecast per segment individually 2020-2024 (5 years)
- Main/major risks (risk matrix, economic slowdown,), (forecast for next 5 years)
Point out main two having biggest impact on company business
Trade war, eu digital tax, low interest rates
Annual share in various years and in years of crises (dot com, 2008)
- List of Appendices
Goodwill ratios
Company's acquisitions (eg MSFT, skype, nokia, linkedin, github in 2018)

The floor is now open for questions.
The valuation was from 9 December

Useful websites: macrotrends.com; statista.com
- Concluding notes
 - Cost of debt: 4.5%, it is calculated on based on historical data
 - Terminal growth – 3.5% based on IMF (its real rate or nominal real rate -> nominal, than its fair, otherwise you should have to add inflation)
 - Economic cycle and revenues
 - Cash debt position

(CFA Institute RC, 2020, notes from final round in Prague, VŠE, 4.2.2020)



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1.2 Global management services

Big Four are collectively referred companies considered among the largest financial and consulting service providers in the accounting and investment management industry, alphabetically sorted:

- **Deloitte**
- **EY (Formerly known as Ernst & Young)**
- **KPMG**
- **PricewaterhouseCoopers (PwC)**

Example 13- Global management services: case of KPMG

KPMG (2014) related to offered global management services, inter alia, offer five major fields for their customers:

- **Creating and setup**
 - creating tax effective structures,
 - business strategy and modelling,
 - remunerating individuals,
 - accounting structures,
 - project management,
 - investor identification
- **Acquisition and sale of investments**
 - sourcing deal flow and raising debt,
 - financing structures,
 - market research,
 - due diligence,
 - modelling,
 - pricing and negotiation support
- **Business as usual**
 - tax compliance,
 - external audit of managers and their funds,
 - technology strategy and operations,
 - valuations,
 - transfer pricing,
 - regulatory advice,





- governance and internal controls,
- reporting (financial/investor/investment performance reporting),
- operational due diligence,
- pensions assurance services,
- risk management (framework, systems, measurement, analytics),
- business strategy,
- operating model design
- management consulting services,
- internal audit,
- management and operational improvement of underlying real assets

- **Refinancing**

- Tax structuring,
- accounting advice,
- debt capacity,
- model reviews

- **Realisation and exits**

- Repatriations to investors,
- valuations,
- IPOs,
- due diligence,
- tax structuring

(KPMG, 2014)

Example 14- The evolution of information technology

- 1960s: the arrival of semiconductor-based microprocessors creating digital data
- 1970s: mainframe computers, capable of processing larger data in one location
- 1980s: distribution of data via remotely connected mini computers, the arrival of personal computers
- 1990s: the evolution continued to encompass mobile phones
- 2000s: the rise of internet and the evolution of smart phones permitting connectivity

- **5G** (fifth generation, digital tools having impact on business, markets, the economy and society)

(KPMG, 2018)



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**Example 15- Examples of the revolutionary technologies evolving in the latest phase of digital evolution**

- blockchain,
 - machine learning
 - robotic process automation
 - speed, connectivity, insights, transparency and disintermediation,
- are currently reshaping business models in finance.

Other examples (KPMG, 2018):

What the internet did to the music business, digitization will do to finance. It's not about 'if' but 'when'.

"In our due diligence, we use scraping analytics that extract data from websites, social media and prediction markets".

(KPMG, 2018)

Example 16- The evolution of information technology

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- **5G** (fifth generation, digital tools having impact on business, markets, the economy and society)

(KPMG, 2018)



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Example 17- Examples of the revolutionary technologies evolving in the latest phase of digital evolution

- blockchain,
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- robotic process automation
- speed, connectivity, insights, transparency and disintermediation,

are currently reshaping business models in finance.

Other examples (KPMG, 2018):

What the internet did to the music business, digitization will do to finance. It's not about 'if' but 'when'.

"In our due diligence, we use scraping analytics that extract data from websites, social media and prediction markets".

(KPMG, 2018)

Example 18- Alternative investment industry (AIM) is facing disruption

This sector is called also "**alternatives**"

- A. Two key segments in the alternative investment industry most amenable to digitization are:
- **hedge funds**
 - **private equity**
- B. The global survey of KPMG (2018) that covers 19 countries with alternative assets under management (a so-called alternative AIM or simply "alternatives") of US\$2.6 trillion shows that:



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C. Alternatives face disruption, as a consequence of digitization and entering the third phase AI 3.0., the adoption of digital innovations so far has been a matter of more haste, less speed.

For example delivering investible information and insights with the science of analytics:

- descriptive analytics: what happened?
- diagnostic analytics: why did it happen?
- predictive analytics: what will happen?
- prescriptive analytics: how do we react?

Business transformation requires digital leaders who can rewrite the traditional narrative on value creation, shifting up business models from product centricity to client centricity as the alternative investment industry enters the third phase, AI 3.0. However, Business transformation requires digital leaders who can rewrite the traditional narrative on value creation.

(KPMG, 2018)

Example 19- **Alternative investment industry (AIM) is facing disruption**

Eight key digital innovations in investment industry:

- Application programming interfaces
- Big data
- Blockchain
- Cognitive computing and machine learning
- New digital platforms for next-gen investors
- Robo advisors
- Robotic process automation
- Social media

(KPMG, 2018)



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**Example 20- Example of tech giants**

Prominent pioneers like the following three tech giants:

- Amazon,
- Google
- and Uber

(KPMG, 2018)

Example 21- B2B vs B2C

- B2B (Business-to-Business) means trade between businesses rather than to consumers
- B2C (Business-to-Consumer) means directly selling to consumers who are end-users

(KPMG, 2018)

Example 22- Examples of activities in investment value chain

- Front office
 - Algorithmic trading
 - Alpha generation
 - Big data analytics
 - Crowdfunding
 - Due diligence
 - Portfolio risk management
 - Product development and fund governance
 - Research and securities selection
- Middle office
 - Asset gathering via digital platforms for end-clients
 - Client on-boarding
 - CRM that works in a cheaper, faster and better way
 - Intermediary relationships
 - Marketing and branding





- Risk and compliance
- Smart contracts
- Back office
 - Depository
 - Fund accounting
 - Global custody
 - Trade and settlements
 - Transfer agent
 - Valuations of illiquid assets

(KPMG, 2018)



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2 QUANTITATIVE INVESTMENT ANALYSIS - PORTFOLIO PERFORMANCE MEASUREMENT

If we open a professional book from the CFA Institute Investment Series dedicated to **quantitative investment analysis**, we will learn that chapters we may find in it include (Defusco et al., 2015a):

- The Time Value of Money,
- Discounted Cash Flow Applications,
- Statistical Concepts and Market Returns,
- Probability Concepts,
- Common Probability Distributions,
- Sampling and Estimation,
- Hypothesis Testing,
- Correlation and Regression,
- Multiple Regression and Issues in Regression Analysis,
- Time-Series Analysis, and
- Introduction to Multifactor Models.

2.1 The time value of money

The time value of money is the key concept in investment mathematics and relate to relationships between cash flows with different dates. Any valuation model cannot be mastered by investment analysis without understanding of the concept of time value of money (TVM).

The core concept of TVM concerns on equivalence relationships between cash flows occurring on different dates. The required compensation in receiving cash flow,



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for example, in one year rather than now is called **rate of return** (and can be in form of e.g. **interest rate**). (DeFusco et. al., 2015a).

2.1.1 Interest rates

When applying the concept of TVM analysts are frequently working with interest rates. Interest rate can be thought in three ways (DeFusco et al., 2015a):

- **Required rates of return** (considered as minimum rate of return one investor require in order to accept the investment)
- **Discount rates** (is a rate investor uses to find the value today from the future value)
- **Opportunity costs** (is the value investors forgo by choosing different option than another)

Example 23- **Opportunity costs**

- We decided to spend 9 500 USD today
- We therefore lost a chance to obtain an interest in a year, for example we would have forgone earning 3.25 percent on the money per year.
- Our opportunity costs of current consumption therefore would be 5.26 percent p.a. (per annun).

(DeFusco, 2015a)

DeFusco (et al., 2015a) remind that, Economics teaches us that **interest rate are set by supply** (where suppliers of funds are considered investors) **and demand** (and demanders of funds are considered borrowers) **forces in the marketplace**.

The interest rate referred as r can be from the persepective of investors viewed as composed of a:

- Real risk free interest rate
- Inflation premium,
- Default risk premium,
- Liquidity premium,
- Maturity premium,



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, and can be modeled as:

$$r = \text{Real risk free rate} + \text{Inflation premium} + \text{Inflation premium} + \text{Default risk premium} + \text{Liquidity premium} + \text{Maturity premium} \quad (1)$$

Example 24- Premiums requiring returns or compensation for bearing distinct types of risk

Inflation premium

- We take into account investors for expected inflation that is reducing them the purchasing power of a unit of currency the amount of goods and services that they can buy, require compensation for a expected maturity.
- Usually, they use average inflation over expected maturity
- Nominal risk free interest rate = real risk free interest rate + inflation premium
- The nominal risk free interest rate is in many countries referred as interest rate of government short-term debt (nominal risk free rate in that country)
- For example in U.S. 90 days T-bills represents nominal risk free interest rate over that horizon

Default risk premium

- Investor compensates the possibility the borrower will fail to repay a promised payment at the contracted time and in the contracted amount.

Liquidity premium

- Investor compensates the risk of loss relative to an investment's fair value, when the investment needs to be converted quickly to cash
- For example U.S. T-bills do not bear a liquidity premium, while large amount can be sold and bought without affecting the market price

Maturity premium

- Is connected to required compensation of investors for increases sensitivity of market value of debt (fixed income securities for example) to changes in market interest rates as maturity is extended
- A positive maturity premium = The interest rate on longer maturity – on short-term or liquid Treasury debt

(DeFusco et al., 2007; DeFusco et al., 2015a)





2.1.2 The future value of single cash flow

- **Simple interest**

The following formula can be used when determining a relationship between an initial investment or present value (PV) earning a rate of return (r) and a future value (FV) in a given time:

$$FV = PV * (1 + r * t) \quad (2)$$

, *where*

PV is the initial value of the capital (deposit, principal, initial amount of loan)

r is the interest rate per the interest period (for example if the interest period is semi annual, it is necessary to apply semi annual interest rate, it means p.s.!))

t is the number of interest periods (in case of annual interest period -> number of years, in case of semi annual period -> number of semesters, etc.)

The golden rule in case of interest estimations is that variables must be defined in the same time units! (Defusco et al., 2015a)

Example 25- The future value of single cash flow – simple interest

Simple decursive interest

- interest is calculated only from the initial capital (principal, loan)
- No interests from accumulated interests
- Usually used, when number of interest periods is less than 1
- The principle of arithmetic sequence is applied
- Principal is the amount of funds originally invested
- **Interest period** – means how often (the frequency) are interests credited/added to the principal (initial capital, loan)
- Decursive interest means, that interests are added (payed) at the end of the interest period and are estimated from the initial capital (PV).



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- Examples of using decursive interest in practice: this type of interest can be used by banks for saving accounts and term accounts. This type of interest can be used also for loans
- **In terms of time, we distinguish five basic interest periods:**
 - p.a. (per annum) annual = once a year
 - p.s. (per semestre) semi annually = every 6 months
 - p.q. (per quartale) quarterly = each 3 months
 - p.m. (per mensem) monthly = each 1 month
 - p.d. (per diem) daily = each 1 day

(DeFusco et al., 2007; DeFusco et al., 2015a)

Example 26- The future value of single cash flow – simple interest

Simple decursive interest

- We deposit 100 000 CZK on our saving account with the annual interest rate of 4% p.a. and semi annual (semester) interest period. How much money we will have in 2 months, if we consider 20 % income tax rate?
- Solution:

$r = 4\% \text{ p.a.} \rightarrow 2\% \text{ p.s.} \rightarrow \text{considering taxes} \rightarrow 2(1-0,20) = 1,60\% \text{ p.s.} \rightarrow 0,016 \text{ p.s.}$

$t = 2 \text{ months} = 1/3 \text{ semester}$

$PV = 100\,000, FV = ?$

$$FV = PV * (1 + r * t)$$

$$FV = 100\,000 * \left(1 + 0,016 * \frac{1}{3}\right)$$

$FV = 100\,533,33 \text{ CZK}$ (where 534 CZK are interests)

- Answer: **We can withdraw 100 533,33 CZK in 2 months.**
Where 534 CZK interest you earned from the original investment is known as simple interest (the interest rate times the principal).

(DeFusco et al., 2007; DeFusco et al., 2015a)



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**Example 27- The future value of single cash flow – simple interest, discounting****Simple decursive interest**

- **What amount should be deposited on the saving account in order to obtain 1 000 CZK in 3 months, considering 4 % annal interest rate, quarterly interest period an 15 % income tax rate?**
- **Solution:**

$r = 4\% \text{ p.a.} \rightarrow 1\% \text{ p.q} \rightarrow \text{considering taxes} \rightarrow 0,01(1-0,15) = 0,0085 \text{ p.q.}$

$t = 3 \text{ months} \rightarrow 1 \text{ quarter}$

$FV = 1000 \text{ CZK, } PV = ? \text{ (CZK)}$

$$PV = \frac{FV}{(1+r*t)}$$

$$PV = \frac{1000}{(1+0,0085*1)} = \mathbf{991,57 \text{ Kč}}$$

- **Answer: 992 CZK is the amount that should be deposited.**

(DeFusco et al., 2007; DeFusco et al., 2015a)

- **Compounded interest**

The following formula can be used when determining a relationship between an initial investment or present value (PV) earning a rate of return (r) and a future value (FV) in a given time:

$$FV = PV * (1 + r)^t \quad (3)$$

, *where*

PV is the initial value of the capital (deposit, principal, initial amount of loan)

r is the interest rate per the interest period (for example if the interest period is semi annual, it is necessary to apply semi annual interest rate, it means p.s.!))

t is the number of interest periods (in case of annual interest period \rightarrow number of years, in case of semi annual period \rightarrow number of semesters, etc.)

The compounded interest earned on the initial investment is more powerful, compared to simple interest, while:



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- the simple interest is fixed in size from period to period, for a given interest rate
- the compounded interest earned on **reinvested interest grows in size each period** (Defusco et al., 2015a)

, implying the simple interest is not taking reinvested interests into account and is estimated only from the initial capital, compared to compounded interest.

And as was aforementioned, **the golden rule in case of interest estimations is that variables must be defined in the same time units!** (Defusco et al., 2015a)

Example 28- The future value of single cash flow – variables in the same time units

- If number of interest (compounding) period (t) is in month
- Then interest rate (r) has to be the one-month interest rate, unannualized

(DeFusco et al., 2007; DeFusco et al., 2015a)

Example 29- The future value of single cash flow – compounded interest

- **How much we will have on our saving account in 4 years, if we deposited 10 000 CZK, connected with quarterly interest period, 2 % p.a interest rate and 15 % income tax rate.**
- **Solution:**
 $r = 2\% \text{ p.a.} > 0,5\% \text{ p.q} \rightarrow \text{considering taxes } 0,005 * (1 - 0,15) = 0,00425 \text{ p.q}$
 $t = 16 \text{ quarters, } PV = 10\,000 \text{ CZK, } FV = ?$
$$FV = PV * (1 + r)^t = 10\,000 * (1 + 0,00425)^{16} = 10\,702 \text{ CZK}$$
- **Answer: We will have 10 702 CZK after 4 months. Where 702 CZK interest you earned from the original investment is known as compounded interest (the interest rate times the principal).**

(Šoba and Širůček, 2017)



**Example 30- The future value of single cash flow – compounded interest**

- **How much we will have in 4 years, if we deposit 6 000 CZK at the beginning of the first year, and at the end of the first year we deposit another 4 000 CZK, interest rate is 2 % p.a., the interest period is quarterly, the income tax is 15 %.**
- **Solution:**
 $r = 2\% \text{ p.a.} = 0,5\% \text{ p.q.} \rightarrow \text{considering taxes} \rightarrow 0,005 * (1 - 0,15) = 0,00425 \text{ p.q.}, t_1 = 16$
 $q, t_2 = 12q$

$$FV = PV * (1 + r)^t = 6000 * (1 + 0,00425)^{16} + 4000 * (1 + 0,00425)^{12} = 10\,630 \text{ CZK}$$
- **Answer: We will have 10 630 CZK in 4 years.**

(Šoba and Širůček, 2017)

Example 31- The future value of single cash flow – compounded interest

- **We have deposited 20 000 CZK at the beginning of the 2 year saving period, and at the end of this period we have 22 000 CZK. What gross annual interest rate was agreed with a bank, considering quarterly interest period and 20 % income tax rate?**
- **Solution:**

$$r = \sqrt[t]{\frac{FV}{PV}} - 1 = \sqrt[8]{\frac{22000}{20000}} - 1 = 0,0120 \text{ p.q.} = 1,2\% \text{ p.q.} = 4,8\% \text{ p.a.}$$

gross interest rate mentioned in the contract (before taxes)

$$r_{\text{net}} = r_{\text{gross}}(1 - t) \rightarrow r_{\text{gross}} = \frac{0,048}{(1 - 0,20)} = 0,06 \text{ p.a.} = 6\% \text{ p.a.}$$
- **Answer: The interest rate written initially in the contract valid for the 2 year period of saving was 6 % p.a.**

(Šoba and Širůček, 2017)



**Example 32- The future value of single cash flow – compounded interest**

For how many years we would have to wait to have 15 000 CZK from the initially deposited 10 000 CZK, using the saving account with a) semi annual interest period b) quarterly interest period; assuming 5 % p.a. interest rate and 20 % income tax rate?

- Solution:

$$t = \frac{\ln FV - \ln PV}{\ln(1 + r)} = \frac{\ln 15\,000 - \ln 10\,000}{\ln(1 + \frac{0,05}{2} * (1 - 0,2))} = 20,48 \text{ semesters} \rightarrow 10,24 \text{ years}$$

$$t = \frac{\ln FV - \ln PV}{\ln(1 + r)} = \frac{\ln 15\,000 - \ln 10\,000}{\ln(1 + \frac{0,05}{4} * (1 - 0,2))} = 40,75 \text{ quarteres} \rightarrow 10,19 \text{ years}$$

- Answer: We would have to wait for a) 10,24 years, b) 10,19 years.

(Šoba and Širůček, 2017)

- Continuous interest

If the number of compounding periods per year becomes infinite, we are using a term compound continuously, that can be expressed with the following formula (limiting value of the future value factor for number of interest rate periods (m) $\rightarrow \infty$, meaning infinitely many compounding periods per year) (DeFusco et. al, 2015a):

$$FV = PV * e^{r*t} \quad (4)$$

, where

FV future value of the capital

PV present value of the capital

r interest rate per interest period -> “annual average”

t number of years

e Euler constant (the base of the natural logarithm, the transcendental number $e \approx 2.7182818$ raised to the power $r*t$; most calculator have the function e^x)

The formula for calculating the effective interest rates exist also for the continuous interest : $r_e = e^r - 1$



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(DeFusco et al., 2015a, Šoba and Širůček, 2017)

Example 33- The future value of single cash flow – continuous compounding

What price can we expect in 120 months, if the long term average change is approximately 5 %? The currently stock price is 1 000 CZK.

- Solution:

$$FV = PV * e^{r*t} = 1\,000 * e^{0,05*10} = 1\,649 \text{ CZK}$$

- Answer: The price of the stock can be around 1 649 CZK in 120 months.

(Šoba and Širůček, 2017)

Effective annual rate (EAR, r_e)

Effective annual rate (EAR) in general it reflects a rate of return of the deposited money applying compounding (in the case of simple interest it is nonsense to estimate effective interest rate, because there are no interests from interests) It enables to compare interest rates with the different frequency of payment (adding) of interest rates (ie with different interest periods) in the same given period (usually per year $r_e \rightarrow$ p.a.).

$$r_e = (1 + r)^p - 1 \quad (5)$$

, where

r_e effective annual rate (EAR)

r interest rate per interest rate period

p number of interest rate periods per year

Example 34- The future value of single cash flow – EAR

Which of the following variants is more beneficial for the depositor? A) The deposit with 2 % p.a. interest rate and a semi annual interest rate period. B) The deposit connected with 2 % p.a. interest rate and a monthly interest period.



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- Solution:

$$Var.A: r_e = (1 + r)^p - 1 = \left(1 + \frac{0,02}{2}\right)^2 - 1 \rightarrow 2,01 \% p. a.$$

$$Var.B: r_e = (1 + r)^p - 1 = \left(1 + \frac{0,02}{12}\right)^{12} - 1 \rightarrow 2,02 \% p. a.$$

- Answer: The more frequent payment of interests leads to a higher return, thereby for the saver is obviously better the variant B.

(Šoba and Širůček, 2017)

2.1.3 The future value of series of cash flows

In the previous section we considered single cash flow, while now we consider series of cash flow, than can be even or uneven. Based on Defusco et al. (2015a) the basic terms commonly used when valuing cash flows distributed over many time periods are the following:

- **An annuity** (is a finite set of level sequential cash flow, in other words it is a regularly deposited or invested amount of money (regular deposit))
- **An ordinary annuity** (has a first cash flow that occurs one period from now)
- **An annuity due** (has a first cash flow that occurs immediately)
- **A perpetuity** (a perpetual annuity, is a set of level never-ending sequential cash flows, with the first C-F occurring one period from now).

There are three possible estimation applications:

- **Future value of annuity (regular deposits saving, regular investment investing)**

E.g. we can ask: How much will I have in 10 years, if I monthly deposit 1 000 CZK?

- **Present value of annuity (pensions, rents)**

E.g.: What amount is necessary to deposit now, in order to obtain monthly for 5 years a rent of 2 500 CZK.

- **The combination of regular deposits savings and pensions**



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(Šoba and Širůček, 2017)

- **The future value of annuity**

The basic identity (uses the same principles of simple and compound interest)

$$S = a * m * \left(1 + \frac{m+1}{2*m} * r\right) * \frac{(1+r)^n - 1}{r} \quad (6)$$

, where

S future value of annuity (ie. the total saved- up amount)

a the amount of annuity (for example mortgage payment, regular deposit)

m number of annuitis (deposits) per year

r interest rate per interest period (or adjusted for taxes)

n number of interes periods ($n > 1$)

the amount saved is dependent on the size of the regular deposit (a , annuity), the number of deposits (annuities) per interest period (m), the interest rate per the interest period (r), and the number of interest periods (n).

$m \pm 1$: $m + 1$ of annuity due, $m - 1$ of ordinary annuity

(Šoba and Širůček, 2017)

Example 35- The future value of series of cash flows – ordinary annuity and annuity due

How much we will have in 5 years if we regularly save at the end of each month 1 000 CZK, at 3 % p.a. interest rate and semi annual interest period? The income tax rate is 15 % p.a. Calculate it also for the case of regularly saving the annuity at the beginning of each month.

- **Solution:**

*m 6 deposits per semester, $n = 5 * 2$ semesters*

Annuity due

$$S = a * m * \left(1 + \frac{m+1}{2*m} * r\right) * \frac{(1+r)^n - 1}{r} = 1000 * 6 * \left(1 + \frac{6+1}{2*6} * \frac{0,03*(1-0,15)}{2}\right) * \frac{\left(1 + \frac{0,03*(1-0,15)}{2}\right)^{10} - 1}{\frac{0,03*(1-0,15)}{2}} = 64\,035 \text{ CZK}$$



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*Ordinary annuity*

$$= 1000 * 6 * \left(1 + \frac{6-1}{2*6} * \frac{0,03*(1-0,15)}{2}\right) * \frac{\left(1 + \frac{0,03*(1-0,15)}{2}\right)^{10} - 1}{\frac{0,03*(1-0,15)}{2}} 63\,900 \text{ CZK}$$

(Šoba and Širůček, 2017)

- **The present value of annuity**

The basic identity (uses the same principles of simple and compound interest)

$$D = a * m * \left(1 + \frac{m+1}{2*m} * r\right) * \frac{1-v^n}{r} * v^k \quad (7)$$

, where

D the present value of annuity (of all pensions, rents in the future)

a the amount of annuity (a rent, a pension)

m the number of payments of annuities (rents) per one interest period

r the interest rate per one interest period (or net income interest rate after income tax rate)

n the total number of interest periods

k the number of interest periods of deferred payments of annuities (pensions, rents)

v discount faktor (denominator), estimated as $\frac{1}{1+r}$, where (r) is the interest rate (or net interest rate) per the

interest period. It is highly recommended to use at least 4 decimal places, in order to obtain accurate results. If payment of annuities is deferred ($k > 0$), it means if the saved up amount is being still deposited on the banking account without depositing or paying annuities, we have to consider separately this deferred period based on the valid interest period in this period (r, k). Simply said interest period in this deferred period can vary from the period of payment of annuities. (eg. the period of paying annuities based on monthly interest period, deffered period based on yearly interest period)

Obviously, the key role plays the interest period, to which all the variables (r), (m), (n) a (k) should be adjusted!

$m \pm 1$: $m + 1$ of annuity due, $m - 1$ of ordinary annuity



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k : The pensions, rents can be ordinary or due, and furthermore:

- **Immediately payed** when annuities are being paid immediately ($k=0$) $\rightarrow (v^k=1)$

For example today we deposit 300 000 CZK and immediately we begin to withdraw an annuity (pensions, rents) monthly at the end of each month for a certain length of period of time. ($k > 0$)

- **Deferred** when there is a delay in the beginning of the payment of annuities

For example we today deposit 300 000 CZK, and after the two years the deposit is being accrued, and after that we begin to receive regularly the annuities.

Immediately payed or deferred pensions, rents, can be furthermore:

- **Temporary pensions** limited payment of annuities (pensions, rents) ($n < \infty$)
- **Perpetual pensions** unlimited payments of annuities (pensions, rents) ($n = \infty$)

$$D = a * m * \left(1 + \frac{m+1}{2*m} * r\right) * \frac{1}{r} * v^k \quad (7)$$

Perpetual annuities are widely used in valuation of securities (eg in the case of dividend discount model, where the holding of securities is assumed as unlimited, while the stocks are generally without maturity and to unlimited payment of dividends are therefore applied principles of perpetual annuities).

(Šoba and Širůček, 2017)

Example 36- The future value of series of cash flows – present value of annuity

What amount we have to deposit today, in order to receive regularly each end of the year a rent of 1 000 CZK, in the following 3 years? We assume 10 % p.a. interest rate and annual interest period. Estimate the results with using the basic identity (formula) based on the principle of annuities.

- Solution:

Not deferred rents because annuities are payed immediately without a deferral, therefore $v^k = v^0 = 1$ and we assume only a period of payment of annuities.

$r=10\%$ p.a. = 0,1 p.a., $m= 1$ payment of pension per year, $n= 3$ years, $a= 1\,000$ CZK (ordinary $\rightarrow m-1$)

$$D = a * m * \left(1 + \frac{m+1}{2*m} * r\right) * \frac{1 - v^n}{r} * v^k$$





$$D = 1\,000 * 1 * \left(1 + \frac{1-1}{2*1} * 0,10\right) * \frac{1 - \left(\frac{1}{1+0,10}\right)^3}{0,10} * 1$$

$$D = 1\,000 * 1 * (1 + 0) * \frac{1 - \left(\frac{1}{1+0,10}\right)^3}{0,10} * 1 = 2\,487 \text{ CZK}$$

- **Answer: We have to deposit today 2 487 CZK.**

(Šoba and Širůček, 2017)

Example 37- The future value of series of cash flows – present value of annuity, deferred pension

- **What amount Mr Novak has to deposit today, in order to receive regularly after 2 years at the end of each month the annuity of 2 500 Kč, for 3 let? The bank account is tied with 3 % p.a. interest rate and semi annual interest period in the first two years and with 4 % p.a. interest rate and quarterly interest period in the next years. The 15 % income tax rate has to be considered.**
- **Solution:**

The deferred period is a 2 year period.

$$r = 4/4/100 * (1 - 0,15) = 0,0085$$

$$D = a * m * \left(1 + \frac{m \pm 1}{2 * m} * r\right) * \frac{1 - v^n}{r} * v^k$$

$$D = 2500 * 3 * \left(1 + \frac{3-1}{2*3} * 0,0085\right) * \frac{1 - \left(\frac{1}{1,0085}\right)^{12}}{0,0085} * \left(\frac{1}{1 + 0,01275}\right)^4$$

$$= 81\,237 \text{ CZK}$$

$$D = 1\,000 * 1 * \left(1 + \frac{1-1}{2*1} * 0,10\right) * \frac{1 - \left(\frac{1}{1+0,10}\right)^3}{0,10} * 1$$

$$D = 1\,000 * 1 * (1 + 0) * \frac{1 - \left(\frac{1}{1+0,10}\right)^3}{0,10} * 1 = 2\,487 \text{ CZK}$$

- **Answer: Mr. Novák has to deposit 81 237 Kč.**



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(Šoba and Širůček, 2017)

Example 38- The future value of series of cash flows – present value of annuity, deferred pension

- We deposited 150 000 Kč into our bank account. For how long time we can receive regularly 5 000 CZK at the beginning of each month? We assume 6 % p.a. interest rate, semi annual interest period and 20 % tax rate.
- Solution:

$$D = a * m * \left(1 + \frac{m \pm 1}{2 * m} * r\right) * \frac{1 - v^n}{r} * v^k$$

$$150\,000 = 5\,000 * 6 * \left(1 + \frac{6 + 1}{2 * 6} * 0,024\right) * \frac{1 - \left(\frac{1}{1,024}\right)^n}{0,024} * 1$$

$$\ln(0,88166) = \ln\left(\frac{1}{1,024}\right)^n \rightarrow \ln(0,88166) = n * \ln\left(\frac{1}{1,024}\right)$$

$$n = \frac{\ln(0,88166)}{\ln\left(\frac{1}{1,024}\right)} = 5,31 \text{ semesters} = 31,86 \text{ months}$$

= 32 payment of pensions

32th annuity (payment) will approximately be $0,86 * 5\,000 = 4\,300$ CZK

- **Answer: We can withdraw after 32 months, ie after 2 years and 8 months.**

(Šoba and Širůček, 2017)

2.2 Discounted cash flow applications

For the fundamental problem of valuing investments applicable to corporate investment (in particular to capital budgeting problems) are considered as essential (DeFusco et al., 2017):

- concepts of **present value**,



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- **net present value (NPV)**, and
- **internal rate of return (IRR)**

2.2.1 The net present value (NPV)

$$NPV = PVCF - C_1 \quad (7)$$

, where

PVCF is present value of future cash flows $PVCF = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \frac{CF_4}{(1+r)^4} + \dots + \frac{CF_t}{(1+r)^t}$

r is discounted rate (required rate of return)

C_1 is an initial capital expenditure

The formula is also known as $NPV = \sum \frac{CF_n}{(1+r)^n}$

The net present value (NPV) of a project is the present value of its cash inflows minus present value of its cash outflows (DeFusco et al., 2017, p.64). NPV is simply a sum of the present values of all the cash flows associated with the investment, where inflows are signed positive

The important rule of NPV for decision making is to

- Accept all projects with positive NPV
- If project are mutually exclusive, to accept the project with the higher positive NPV
- With mutually exclusive projects we rely on the NPV rule.

and outflows are signed negative.



**Example 39- The present value of cash flows (PVCF)**

- How much are you willing to pay for an investment if you consider the following future cash flows – in the first year (100 CZK), in the second year (150 CZK), and in the third year (2000 CZK including the selling price). The required rate of return is 6 % p.a.

- Solution:

$$PVCF = \frac{100}{(1+0,06)^1} + \frac{150}{(1+0,06)^2} + \frac{2000}{(1+0,06)^3} = 1\,907 \text{ CZK}$$

- Answer: Investor is willing to pay 1907 CZK as a maximum price.

(Šoba and Širůček, 2017)

Example 40- The net present value of cash flows (NPV)

Investor has to decide between two investment projects X and Y. CFs of investment X are forecasted as -900 000 CZK (phase 0), +320 000 CZK (phase 1), +340 000 CZK (phase 2), +390 000 CZK (phase 3), +420 000 CZK (phase 4). And CFs of investment Y are forecasted as - 1 000 000 CZK (phase 0), +220 000 CZK (phase 1), +220 000 CZK (phase 2), +310 000 CZK (phase 3), +430 000 CZK (phase 4). The discount rate is 5 % for investment X, while in case of investment Y investor requires less 4 %. Which investment would investor favor based on NPV rule?

Solution:

5%	X	- 900 000	+ 320 000	+ 340 000	+ 390 000	+ 420 000
4%	Y	- 1 000 000	+ 220 000	+ 220 000	+ 310 000	+ 430 000

Formula NPV for NPV_x and NPV_y

$$NPV_x = \sum \frac{CF_n}{(1+r)^n} = \frac{-900000}{(1+0,05)^0} + \frac{320000}{(1+0,05)^1} + \frac{340000}{(1+0,05)^2} +$$





$$+ \frac{390000}{(1 + 0,05)^3} + \frac{420000}{(1 + 0,05)^4} = 395\,584 \text{ CZK}$$

$$NPV_y = \sum \frac{CF_n}{(1 + r)^n} = \frac{-100000}{(1 + 0,04)^0} + \frac{220000}{(1 + 0,04)^1} + \frac{220000}{(1 + 0,04)^2} + \frac{310000}{(1 + 0,04)^3} + \frac{430000}{(1 + 0,04)^4} = 58\,095 \text{ CZK}$$

- **Answer:** Based on calculation NPV investor would favour investment X.

Example 41- The net present value of cash flows (NPV)

Waldrup Industries is considering a proposal for a joint venture that will require an investment of 13 million USD. At the end of the fifth year, Waldrup's joint venture partner will buy out Waldrup's interest for 10 million USD. Waldrup's CFO has estimated that the appropriate discount rate for this proposal is 12 percent. The expected cash flows are given below.

Year	Cash Flow (USD)
0	-13 000 000
1	3 000 000
2	3 000 000
3	3 000 000
4	3 000 000
5	10 000 000

Calculate the proposal's NPV and make a recommendation to the CFO (chief financial officer) concerning whether Waldrup should enter into this joint venture.

- **Solution:**

$$NPV_x = \sum \frac{CF_n}{(1 + r)^n} = \frac{-13\,000\,000}{(1 + 0,12)^0} + \frac{3\,000\,000}{(1 + 0,12)^1} + \frac{3\,000\,000}{(1 + 0,12)^2} +$$





$$+ \frac{3\,000\,000}{(1 + 0,12)^3} + \frac{3\,000\,000}{(1 + 0,12)^4} + \frac{10\,000\,000}{(1 + 0,12)^5} = 1\,786\,317$$

PV of the inflows is 14 786 317 USD

Present expenditure is 13 000 000 USD

Therefore NPV is 1 786 317 USD.

- **Answer: Waldrup should undertake this project because it has a positive NPV.**

(Defusco et al., 2015b)

2.2.2 The internal rate of return (IRR)

$$0 = NPV \Rightarrow 0 = PVCF - C_1 \Rightarrow 0 = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \dots + \frac{CF_t}{(1+r)^t} - C_N \quad (8)$$

The internal rate of return (IRR) is the discount rate that makes NPV equal to 0 and can be interpreted as an expected compound return only when all interim cash flows can be reinvested at the IRR and the investment is maintained to maturity. (DeFusco et al., 2017, p.64).

$$NPV = \sum \frac{CF_n}{(1+r)^n} = 0 \quad (9)$$

The important rule of IRR for decision making is to (Defusco et al., 2017):

- Accept all projects with an internal rate of return exceeding the required rate of return
- IRR can not be interpreted as achievable rates of return, because this ranking is not considering external factors and the estimation of IRR generally assumes reinvestment at the IRR.
- When dealing with mutually exclusive projects use NPV rule when IRR rule and NPV rule conflict. While NPV rule's assumption about reinvestment rates is more realistic and economically relevant incorporating market-determined opportunity cost of capital in as a discount rate. While, IRR is not affected by external interest rate or discount rate, because a project's cash flows alone determine the IRR





(DeFusco et al., 2017, p.51).

Example 42- Internal rate of return

- **The investment into security is connected with following cash flow in following years: - 900 000 CZK (phase 0), 250 000 CZK (phase 1), 850 000 CZK (phase 2). Calculate IRR manually.**

- **Solution:**

Note. We determine two interest rates to obtain one NPV positive and one NPV negative (higher interest rate) in order to calculate IRR

$$IRR = r_p + \frac{NPV_p}{(NPV_p - NPV_n)} + (r_n - r_p) \quad (10)$$

We need to estimate NPV_p (positive) and NPV_n (negative). In order to do that it is necessary to find IRR (Internal Rate of Return) which has value only when a sum of positive NPV and negative NPV equals zero.

Step 1: Let us choose interest rates in order to get positive (+) NPV and negative (-) NPV the sum of which eventually gives zero:

$$i = 5\% \Rightarrow r = 0,05: \quad NPV_1 = \frac{-900}{(1+0,05)^0} + \frac{250}{(1+0,05)^1} + \frac{850}{(1+0,05)^2} = 109\,070 \text{ CZK (+)}$$

$$i = 20\% \Rightarrow r = 0,2: \quad NPV_2 = \frac{-900}{(1+0,2)^0} + \frac{250}{(1+0,2)^1} + \frac{850}{(1+0,2)^2} = -101\,389 \text{ CZK (-)}$$

If we calculate NPV₁ and NPV₂ the result will be close to zero. If we want to get 0 we should more

accurate choose interest rates.

Step 2:

$$YTM = r_p + \frac{NPV_p}{(NPV_p - NPV_n)} + (r_n - r_p) = 5 + \frac{109\,070}{109\,070 - (-101\,389)} \times (20 - 5) =$$

12,77%

- **Answer: Internal rate of return is 17.77 %.** This IRR for this project can be compared with IRR of other projects.



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**Example 43- Internal rate of return**

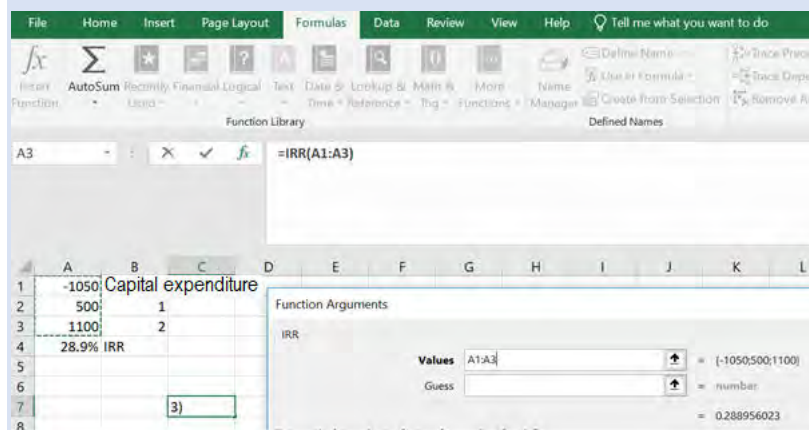
Determine equation for IRR for investments with the following expected cash flows, 500 CZK in the first year and 1 100 in the second year. The current market price is 1 050 CZK.

- Solution:

$$1\,050 = \frac{500}{(1+IRR)^1} + \frac{1100}{(1+IRR)^2}; \quad IRR=? (\%)$$

- Solution in MS Excel using IRR function

Formulas/Financial/IRR



- **Answer: Internal rate of return is 28.9 %.** This IRR for this project can be compared with IRR of other projects.

Example 44- Internal rate of return

- **Waldrup Industries has committed to investing 5 500 000 USD in a project with expected cash flows of 1 000 000 USD at the end of Year 1; 1 500 000 at the end of Year 4, and 7 000 000 USD at the end of Year 5. Demonstrate that the internal rate of return is 13.51 percent.**



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State how the internal rate of return of the investment would change if Waldrup's opportunity costs of capital were to increase by 5 percentage points.

- **Solution:**

Rule:
$$NPV = CF_0 + \frac{CF_1}{(1+IRR)^1} + \frac{CF_2}{(1+IRR)^2} + \dots + \frac{CF_n}{(1+IRR)^n} = 0$$

We need to show IRR of 13.51 percent equals zero the investment's NPV:

$$NPV = -5\,500\,000 + \frac{1\,000\,000}{(1+0,1351)^1} + \frac{1\,500\,000}{(1+0,1351)^4} + \frac{7\,000\,000}{(1+0,1351)^5} = -5\,500\,000 + 5\,499\,266 = -734 \text{ USD}$$

- **Answer: The amount of NPV negligibly differs from 0. The IRR is unaffected by any change in any external rate, including the increase in Waldrup's opportunity cost of capital.**

(DeFusco et al., 2015b)

2.2.3 Portfolio return measurement

The field of performance measurement is of great importance of investment managers and investors while it is widely considered by professional analysts as it is the foundation for all further analysis.

Investor needs to calculate returns in a logical and consistent manner (accurate **performance measurement**) in order to **assess the success of investments**. The term performance evaluation can be referred also as performance appraisal (DeFusco et al., 2015a).

The accurate measurement of portfolio return is essential when one can judge the work of portfolio managers.

- **Holding period returns (HPR)**

This fundamental concept is based on the fact investor earns the return over a specified holding period



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- For **one cash payment at the end of the holding period**:

$$HPR = (P_1 - P_0 + D_1)/P_0 \quad (10)$$

, where

P_1 is the price received at the end of the holding period

D_1 is cash paid by the investment at the end of the holding period

- For measuring performance over **many periods**, or **portfolio is subjected to additions and withdrawals**, the performance measurement is a challenging task. Two approaches are usually used
- **Money-weighted rate of return** using principles of IRR estimation.
- **Time-weighted rate of return**

- **Money-weighted rate of return**

This concept is based on IRR calculation, while IRR is called money-weighted rate of return (in the United States it is referred as the dollar-weighted return) in investment management applications (DeFusco et al., 2015a).

Example 45- Money-weighted rate of return

An investment covers a two-year horizon. At time $t=0$, an investor buys one share at 200 USD. At time $t=1$ the investor purchases an additional share at 225 USD. At the end of the year 2, $t=2$, the investor sells shares for 235 USD each. During both years, the stock pays a per-share dividend of 5 USD. The $t = 1$ dividend is not reinvested. Determine the money-weighted return on this portfolio

Solution:

- The money-weighted return on this portfolio is its internal rate of return for the two-year period.



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- While we consider the aforementioned rule of IRR saying it is the rate, for which present value of cash inflows minus the present value of the cash outflows equal 0:

$$PV(\text{outflows}) = PV(\text{inflows})$$

$$200 + \frac{225}{(1+r)^1} = \frac{5}{(1+r)^1} + \frac{480}{(1+r)^2}$$

In other words we are solving the quadratic equation: $480x^2 - 220x - 200 = 0$, with $x = 1/(1+r)$

Solution in MS Excel:

	A	B	C	D	E	F
1	-200 Initial capital expenditure					
2	-220	1				
3	480	2				
4	9.39%	IRR				

Answer: The spreadsheet's IRR function find that the money-weighted rate of return is 9.39 percent.

The portfolio in the first year generated a one-period holding period return of $(225-200+5)/200 = 15\%$

In the second year the portfolio produced a holding period return of $(470-450+10)/450 = 6.67\%$





The mean holding period return was $(15+6.67)/2 = 10.84\%$

It can be seen that the money-weighted rate of return put higher emphasis on the second year's relatively poor performance (6.67 %), compared to the first year's relatively good performance (15%), as more money was invested in the second year than in the first year.

The serious drawback, however, of the money-weighted rate of return from the point of view of a tool for evaluating investment managers, stems from the general principle of evaluation that evaluation tool should isolate the effects of the investment manager's actions to enable judging an investment manager only on the basis of his or her own actions. But in this case investment manager's clients determine when money is given to the investment manager and how much money is given.

Time-weighted rate of return can be the effective in that respect.

(DeFusco et al., 2015a)

- **Time-weighted rate of return**

This concept is not sensitive to the withdrawals and additions of funds and is being the preferred performance measure in the investment management practice (DeFusco et al., 2015a).

The time weighted rate of return:

- Measures the compound rate of growth of 1 USD initially invested in the portfolio over a stated measurement period
- Is not affected by cash withdrawals or additions to the portfolio, compared to money-weighted rate of return (not sensitive to amount of withdrawals and additions to the portfolio)
- Returns are averaged over time (not sensitive to timing)

(DeFusco et al., 2015a).

Steps to estimate time weighted rate of return:

- Before any significant addition or withdrawal of funds, price the portfolio and break the overall evaluation period into subperiods based on the dates of cash inflows and outflows. The more frequent the valuation before any important addition



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or withdrawals, the more accurate the approximation (daily valuation is common-place)

- Calculate the holding period return on the portfolio for each subperiod
- Compound holding period returns to obtain an annual rate of return for the year (the time-weighted rate of return) and if the investment is for more than 1 year, taking the geometric mean of the annual return to obtain the time-weighted rate of return over that measurement period is a good practice.

(DeFusco et al., 2015a).

It was mentioned that daily valuation of portfolio is appropriate practice. To compute the time-weighted return for the year, each **day's holding period return** can be estimated as:

$$r_t = \frac{MVE_t - MVB_t}{MVB_t} \quad (11)$$

, where

r_t daily holding period return

MVE_t is the market value at the end of the day t

MVB_t is the market value at the beginning of the day t

(DeFusco et al., 2015a).

Then, 365 such daily returns ($r_1 \dots r_{365}$) is needed to compute to obtain the **annual return** for the year by linking (compounding) daily holding period returns:

$$(1 + r_1) * (1 + r_1) * \dots * (1 + r_{365}) - 1 \quad (12)$$

The annualized time-weighted return (for each year individually) can be estimated using the geometric mean of N annual returns, as follows:

$$r_{TW} = [(1 + r_1) * (1 + r_1) * \dots * (1 + r_N)]^{\frac{1}{N}} - 1 \quad (13)$$

Example 46- Time-weighted rate of return

An investment covers a two-year horizon. At time t=0, an investor buys one share at 200 USD. At time t=1 the investor purchases and additional share at 225 USD. At the end of the year 2, t=2, the investor sells shares for 235 USD





each. During both years, the stock pays a per-share dividend of 5 USD. The $t = 1$ dividend is not reinvested. Determine the time-weighted return on this portfolio

Solution:

- Portfolio earned returns 15 percent during the first year and 6.67 during the second year
- The portfolio's time-weighted rate of return over an evaluation period of two years can be estimated by taking the geometric mean of the two holding period returns, mirroring the calculation of a compound growth rate

$$(1 + \text{time-weighted return})^2 = (1.15)(1.0667)$$

$$\text{time-weighted return} = \sqrt{(1.15)(1.0667)} - 1 = 10.76\%$$

Answer: The time-weighted return on the portfolio was 10.76 percent.

We can see that it is higher than the money-weighted rate of return. Investment managers find time-weighted rate of returns more meaningful, while it removes effects of giving funds by a client in time. While in case of money-weighted returns, if investor give funds to invest at an unfavorable time the manager's money-weighted return will tend to be depressed and vice versa.

(DeFusco et al., 2015a)

Example 47- Time-weighted rate of return

Strubeck Corporation sponsors a pension plan for its employees. A chief investment officer (CIO) want from you to review the performance of the in-house and Super Trust portfolios over the last four quarters. You have arranged for outflows and inflows to the portfolios to be made at the very beginning of the quarter. The amount invested is the amount each portfolio manager is responsible for investing.

	1Q	2Q	3Q	4Q
--	----	----	----	----



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<i>In house account</i>				
Beginning value	4 000 000	6 000 000	5 775 000	6 720 000
Beginning of period inflow (outflow)	1 000 000	(500 000)	225 000	(600 000)
Amount invested	5 000 000	5 500 000	6 000 000	6 120 000
Ending value	6 000 000	5 775 000	6 720 000	5 508 000
<i>Super trust account</i>				
Beginning value	10 000 000	13 200 000	12 240 000	5 659 200
Beginning of period inflow (outflow)	2 000 000	(1 200 000)	(7 000 000)	(400 000)
Amount invested	12 000 000	12 000 000	5 240 000	5 259 200
Ending value	13 200 000	12 240 000	5 659 200	5 469 568

Calculate the time weighted rate of return for the in-house account.

Calculate the time weighted rate of return for the Super Trust account.

Solution to In house account:

$$1Q \text{ HPR: } (6\,000\,000 - 5\,000\,000) / 5\,000\,000 = 0.20$$

$$2Q \text{ HPR: } (5\,775\,000 - 5\,500\,000) / 5\,500\,000 = 0.05$$

$$3Q \text{ HPR: } (6\,720\,000 - 6\,000\,000) / 6\,000\,000 = 0.12$$

$$4Q \text{ HPR: } (5\,508\,000 - 6\,120\,000) / 6\,120\,000 = -0.10$$

$$(1 + r_1) * (1 + r_2) * ... * (1 + r_4) - 1 = (1+0.20)*(1+0.05)*(1+0.12)*(1-0.10) - 1 = 0.27 = 27 \%$$

Solution to Super Trust account:

$$1Q \text{ HPR: } (13\,200\,000 - 12\,000\,000) / 12\,000\,000 = 0.10$$

$$2Q \text{ HPR: } (12\,240\,000 - 12\,000\,000) / 12\,000\,000 = 0.02$$





$$3Q \text{ HPR: } (5\,659\,200 - 5\,240\,000) / 5\,240\,000 = 0.08$$

$$4Q \text{ HPR: } (5\,469\,568 - 5\,259\,200) / 5\,259\,200 = 0.04$$

$$(1 + r_1) * (1 + r_1) * ... * (1 + r_4) - 1 = (1.10) * (1.02) * (1.08) * (1.04) - 1 = 0.26 = 26 \%$$

Answer: The account managed by Super Trust has a time-weighted rate of return 26 percent, while the time-weighted rate of return for the in-house account was calculated as 27 percent as an annual return. The account managed by Super Trust was relatively poorer.

(DeFusco et al., 2015a)

- **Risk-adjusted performance**

Analysts should take into account and weigh also risk, not only to consider returns. Several important risk-adjusted performance measure might be applied to an investment manager's time-weighted rate of return (for example Sharpe ratio).

Example 48- Risk-adjusted performance

The investor can choose from two stocks (shares), ie stock A or stock B. Which investment will investor choose based on the expected risk-based returns? We assume a risk averse investor.

Return on stock A	Return on stock B	Probability of return occurrence
5 % p.a.	20 % p.a.	0.2
7 % p.a.	8% p.a.	0.3
13 % p.a.	8% p.a.	0.3
15 % p.a.	6% p.a.	0.2





Solution:

$$r_{\text{exp}} = \sum_{i=1}^n r_i * p_i \quad (14)$$

r_{exp} average expected yield (return) of investment

r_i particular expected yield (return)

p_i probability of r_i occurrence

$$\sigma = \sqrt{\sum_{i=1}^n (r_i - r_{\text{exp}})^2 * p_i} \quad (15)$$

σ volatility of investment (SD = standard deviation)

Average expected return (yield)

Stock A: $r_{\text{exp}} = (0,05 * 0,2 + 0,07 * 0,3 + 0,13 * 0,3 + 0,15 * 0,2) =$
10% p. a.

Stock B: $r_{\text{exp}} = (0,2 * 0,2 + 0,08 * 0,3 + 0,08 * 0,3 + 0,06 * 0,2) =$
10% p. a.

Risk (volatility of investment):

Stock A: $\sigma =$

$$\sqrt{(0,10 - 0,05)^2 * 0,2 + (0,10 - 0,07)^2 * 0,3 + (0,10 - 0,13)^2 * 0,3 + (0,10 - 0,15)^2 * 0,2} =$$

3,9 % p. a.

Stock B: $\sigma =$

$$\sqrt{(0,10 - 0,2)^2 * 0,2 + (0,10 - 0,08)^2 * 0,3 + (0,10 - 0,08)^2 * 0,3 + (0,10 - 0,06)^2 * 0,2} =$$

5,06 % p. a.

Return risk ratio:

Stock A = $10:3,9 = 2,56\%$ p.a.

Stock B = $10:5,06 = 1,98\%$ p.a.

-> A better ratio we can see for stock A





Answer: Both investment options have the same average expected return of 10%. Higher volatility is however in case of stock B. When evaluating returns versus risk, a risk averse investor (as the type of return-risk profile) chooses share A.

Do not just watch just revenues, but always after check the volatility! (Sharpe Ratio is eg used in case of mutual funds)

(Šoba and Širůček, 2017)

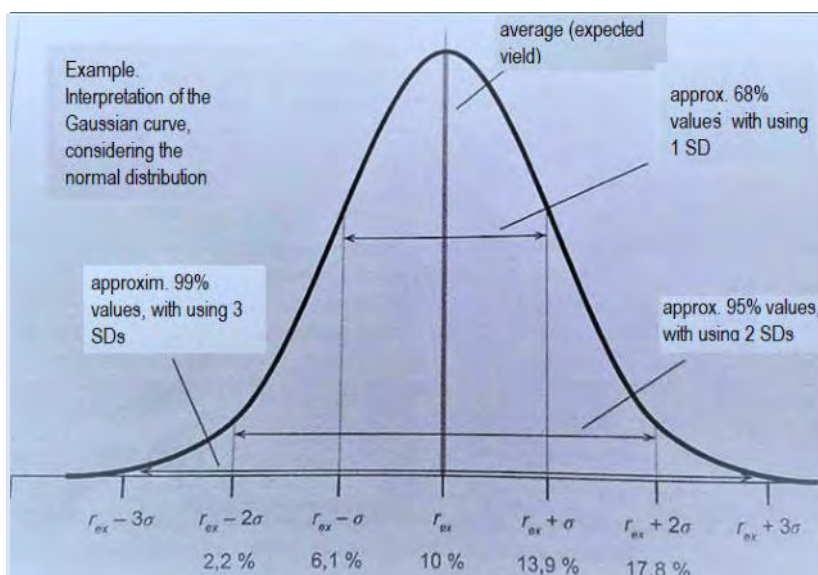
Example 49- Risk-adjusted performance

Interpret the results obtained with the Gaussian curve.

Average expected yield (return): Stock A: 10% p. a. StockB: 10% p. a.

Average risk (volatility): Stock A: = 3,9 % p. a. Stock B: = 5,06 % p. a.

Solution:



Answer: The basic interpretation of the constructed Gaussian curve:

- Most investors achieve approx. the average yield.



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- Number of investors who has made extreme profits is much more less, but roughly is the same as number of investors who has achieved extreme loss.

The normal distribution, on which many financial market models (and not only these) are based, can be simply explained as follows. Most investors achieve a yield approaching the average. Number of investors who has made extreme profits is much more less, but roughly is the same as number of investors who has achieved extreme loss.

Since financial market extremes are very common and are more likely to occur recently, the distribution of returns (yields) does not fully match the normal distribution and causes a drop in probabilities. The 68% probability is so maximal and theoretical (perfect normal distribution of the Gaussian curve) and is actually lower. Still, but we can assume it's over 50%.

(Šoba and Širůček, 2017)



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3 PORTFOLIO PERFORMANCE EVALUATION

The large amounts of money is being managed for other people (investors) by (CFA Program Curriculum, 2018)

- Institutional money managers
- Pension fund managers
- Mutual fund managers

Whether these managers are doing a beneficial job can be an important question. In other words, whether they have actively managed fund better performance than a passively managed portfolio without a manager.

Portfolio performance has two main components:

- **Risk**
- **Return**

, while return maximalization is a common objective, comparing just return of a portfolio with that of benchmark (market, index) is not sufficient. Risk averse investors require compensation for higher risk in the form of higher expected returns.

Based on finance theory in the long run, investors should be compensated with additional mean return at least above the risk-free rate for bearing additional risk, if the risky portfolio is well diversified.

Four ratios are commonly used in performance evaluation and are considered as important **risk-adjusted performance measures**.

- Sharpe Ratio
- Treynor Ratio
- M-Squared (M2)
- Jensen's Alpha

(CFA Program Curriculum, 2018)

3.1 Sharpe ratio

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p} \quad (16)$$



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,where

R_p = the mean return to the portfolio,

R_f = the mean return to a risk-free asset,

σ_p = the standard deviation of return on the portfolio

Sharpe ratio (CFA Program Curriculum, 2018):

- was created by William Sharpe (Sharpe, 1964)
- is also called reward-to-variability ratio
- uses the total risk of the portfolio, not its systematic risk
- the numerator should be positive to give meaningful results

How to interpret Sharpe ratio (CFA Program Curriculum, 2018) ?

- The portfolio with the highest Sharpe Ratio has the best performance
- The portfolio with the lowest Sharpe Ratio has the worst performance

Two important limitations (CFA Program Curriculum, 2018)

- Using the total risk (standard deviation of returns) when only systematic risk is priced
- The ratio itself is not informative (eg 0.2), only if compared to Sharpe ratio of another portfolio

Example 50- Sharpe ratio

Consider the performance of two exchange traded funds. SPDR S&P 500 seeks to track the investment results of the S&P 500 Index (large capitalization US stocks) and iShares Russell 2000 Index seeks to track the investment results of the Russell 2000 Index (small capitalization US stocks). Table below presents the historical arithmetic mean return, along with the historical standard deviation of returns, for annual returns series of these two funds and the US 30-day T-bill during the 2003–2012 period.





Fund/T-Bill	Arithmetic Mean (%)	Standard Deviation of Return (%)
iShares Russell 2000 Index	9.26	22.36
SPDR S&P 500 Index	6.77	19.99
30-day T-bill	1.58	1.78

Sources: finance.yahoo.com and www.federalreserve.gov.

Solution:

$$\text{Sharpe Ratio iShares Russell 2000} = \frac{9.26 - 1.58}{22.36} = 0.34$$

$$\text{Sharpe Ratio SPDR S\&P 500} = \frac{6.77 - 1.58}{19.99} = 0.26$$

Answer: Although US small stocks (iShares Russell 2000 Index) had a higher standard deviation, they performed better than the US large stocks (SPDR S&P 500 Index), as measured by the Sharpe ratio.

(CFA Program Curriculum, 2018)

3.2 Treynor Ratio

$$\text{Treynor Ratio} = \frac{R_p - R_f}{\beta_p} \quad (17)$$

.where

R_p = the mean return to the portfolio,

R_f = the mean return to a risk-free asset,

β_p = beta risk

Treynor Ratio (CFA Program Curriculum, 2018):

- was created by Jack Treynor (Treynor, 1961, 1962)



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- is a simple extension of the Sharpe ratio and resolves its limitation on total risk
- The ratio substitutes beta risk ie systematic risk, for total risk and measures a portfolio's excess returns to the portfolio's beta.
- Numerator and denominator should be positive to give meaningful results

How to interpret *Treynor ratio*? (CFA Program Curriculum, 2018):

- The portfolio with the highest Treynor Ratio has the best performance
- The portfolio with the lowest Treynor Ratio has the worst performance

Limitation (CFA Program Curriculum, 2018):

- The ratio itself is not informative (eg 0.2), onfly if compared of another portfolio
- Similarly like Sharpe ratio we do not whether either of the compared compared is better than the passive portfolio portfolios -> Jensen's alpha and M^2 attempt to resolve this problem

3.3 M-Squared (M^2)

$$M^2 = (R_p - R_f) * \frac{\sigma_m}{\sigma_p} - (R_m - R_f) \quad (18)$$

,where

R_p = the mean return to the portfolio,

R_f = the mean return to a risk-free asset,

σ_m = the standard deviation of return on the market

σ_p = the standard deviation of return on the portfolio

M-Squared (M^2) (CFA Program Curriculum, 2018):

- was created by Franco Modigliani and his granddaughter Leah Modigliani -> therefore: M^2
- gives rankings identical to Sharpe ratio, but in percentage terms easier to interpret and is directly compared with the market return (benchmark)
- By using M^2 , we are able to determine the rank of a portfolio and, if any, of our portfolios beat the market on a risk-adjusted basis.



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- Limitation: is similarly like Sharpe ratio, based on total risk, not beta risk.

How to interpret M^2 ? (CFA Program Curriculum, 2018):

- A portfolio that matches the performance of the market will have an M^2 of zero
- A portfolio that outperforms the market will have an M^2 that is positive.

3.4 Jensen's Alpha

$$\alpha_p = R_p - [R_f + \beta_p * (R_m - R_f)] \quad (19)$$

,where

where R_p = the actual portfolio return

R_f = the average risk-free rate

β_p = beta risk

α_m of the market is zero

Jensen's Alpha (CFA Program Curriculum, 2018):

- is a simple extension of the Sharpe ratio and resolves its limitation on total risk using the beta of the portfolio β_p and the CAPM to quantify the risk-adjusted return of the portfolio (it substitutes beta risk β_p ie systematic risk, for total risk, like Treynor ratio)
- the systematic risk β_p can be measured by estimating the market model, by
- regressing the portfolio's daily return on the market's daily return
- Jensen's alpha α_p is simply a measure of the portfolio's performance relative to the market portfolio, measured as the difference between the actual portfolio return and the calculated risk-adjusted return

How to interpret *Jensen's Alpha ratio*? (CFA Program Curriculum, 2018):



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- If α_p is **positive**, then the portfolio has outperformed the market
- If α_p is **negative**, then the portfolio has underperformed the market
- Jensen's alpha is the maximum amount that we should be willing to pay the manager to manage your money.

Example 51- Jensen's Alpha ratio

Estimated portfolio's (A) alpha is 2 percent and another portfolio's (B) alpha is 5 percent. Quantify which portfolio outperformed and interpret the results to the investor.

- Solution:

$$\alpha_{p(A)} = R_p - [R_f + \beta_p * (R_m - R_f)] = 2 \%$$

$$\alpha_{p(B)} = R_p - [R_f + \beta_p * (R_m - R_f)] = 5 \%$$

by the definition: $\alpha_m = 0 \%$; $\beta_m = 1$;

-> portfolio A outperformed the market by 3 %

-> portfolio B outperformed the market by 5 %

$$\alpha_{p(A)} < \alpha_{p(B)} \rightarrow 5 \% - 2 \% = 3 \%$$

- **Answer: The portfolio B has outperformed the portfolio A by 3 percentage points and the market by 5 percentage points. Jensen's alpha is the maximum amount that the investor should be willing to pay the manager to manage his/her money.**

(CFA Program Curriculum, 2018)



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SUMMARY

To summarize this handbook dedicated to the role of multinational entities in the globalized world in real applications, answer the following discussion question.

Discussion questions

1. Explain the opportunities and challenges facing currently by the investment management industry.
2. Discuss the skills, knowledge and abilities a professional investor must know.
3. Mention the main fields of GBIC® and discuss with real particular examples in the each field.
4. Explain what does “Big four” stands for in auditing industry.
5. Explain major global management services that are being offered by KPMG.
6. Discuss the evolution of the information technology and mention few examples of the revolutionary technologies evolving in the latest phase of digital evolution.
7. Explain what does mean “alternatives”.
8. Distinguish between B2B and B2C.
9. Mention few examples of activities in investment value chain.
10. Mention basic fields that are related to the quantitative investment analysis.
11. Interpret interest rates as required rates of return, discount rates, or opportunity costs
12. Explain an interest rate as the sum of a real risk-free rate and premiums that compensate investors for bearing distinct type of risk
13. Calculate and interpret simple interest, compounded interest, the effective annual rate and continuous interest.
14. Solve time value of money problems for different frequencies of compounding.
15. Calculate and interpret the future value (FV) and the present value (PV) of a single sum of money, an ordinary annuity, an annuity due, a perpetuity (PV only), and a series of unequal cash flows.
16. Calculate and interpret the net present value (NPV) and the internal rate of return (IRR) of an investment
17. Contrast the NPV rule to the IRR rule, and identify problems associated with the IRR rule



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18. Calculate and interpret a holding period return (total return).
19. Calculate and compare the money-weighted and time-weighted rates of return of a portfolio and evaluate the performance of portfolios based on these measures
20. Explain risk-adjusted measures for portfolio evaluation.



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LIST OF ABBREVIATIONS

AI	Artificial intelligence
AUM	Assets under management
B2B	Business-to-Business
B2C	Business-to-Consumer
CAGR	Compound annual growth rate
CBOK	Candidate Body of Knowledge
DCF	Discounted cash flow
EAC	Education Advisory Committee
EAR	Effective annual rate
EBITDA	Earnings before interest, taxes, depreciation and amortization
EVA	Economic value added
FCFE	Free cash flow to equity
FCFF	Free cash flow to firm
FV	Future value
GBIC	Global Body of Investment Knowledge
HPR	Holding period returns
IMF	International monetary fund
IRR	Internal rate of return
NPV	Net present value
p.a.	Annually
p.d.	Daily
p.m.	Monthly
p.q.	Quarterly





p.s.	Semi-annualy
PV	Present value
PVCF	Present value of cash flows
SWOT	Strengths, weaknesses, opportunities, and threats
T-bills	Treasury Bills
TVM	Time value of money



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LIST OF APPENDIXES



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